The National Strategies
Primary

What Works for Children with Mathematical Difficulties?

department for
children, schools and families

What Works for Children with Mathematical Difficulties?

The effectiveness of intervention schemes

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Chapter 1: Focus and intention of this report: context, background and problems to be addressed

Context

The purpose of this report is to update the detailed research report by Dr Ann Dowker published by the Department for Education and Schools (DfES) in June 2004, 'What works for children with mathematical difficulties' (DfES Research Report RR 554). This report examined the status of interventions for children with mathematical difficulties. At that time, there was relatively little work in that area compared, for example, with work on interventions for literacy difficulties (Brooks, 2002). The review reported the evidence concerning the incidence and nature of mathematical difficulties, and the history and current state of interventions for such difficulties. The main conclusion was: 'No two children with arithmetical difficulties are the same. It is important to find out what specific strengths and weaknesses an individual child has; and to investigate particular misconceptions and incorrect strategies that they may have. Interventions should ideally be targeted toward an individual child's particular difficulties. If they are so targeted, then most children may not need very intensive interventions.'

There were relatively few such targeted interventions available at that time, although there had been some work on developing and using such interventions since the 1920s, mainly in America. Individualised programmes known to be in current use at that time in Britain included the Mathematics Recovery programme developed in Australia by Bob Wright and his colleagues (Wright et al, 2000, 2002); Dowker's (2001) pilot Numeracy Recovery programme (since modified and developed as Catch Up Numeracy; and some computerised interventions, such as RM Maths (Earl, 2003). The Department for Education and Skills had introduced the Springboard programme for small groups (Wave 2) and was in the process of piloting its materials for individualised use (Wave 3).

This present report focuses on the subsequent development and use of intervention materials and programmes within the UK since 2004. It does not discuss the earlier history (for which the reader is referred to the earlier review); nor does it discuss work outside the UK. Following both this research review and the publication of the Primary National Strategy's (PNS) guidance and continuing professional development (CPD) materials 'Supporting children with gaps in their mathematical understanding' (DfES 1168-2005G), there has been some development of mathematics intervention programmes by both local authorities (LAs) and national/independent organisations and further use of and data collection for some of the programmes reviewed in 2004.

The aim of this research report will be to inform schools' and LAs' planning and use of Wave 2 and 3 intervention and support for mathematics by providing an independent research review into programmes currently in use which carry a robust evidence base in terms of measurable impact on children's learning and progress.

More exactly, this research report is addressing the following questions:

- What intervention schemes are currently in use in the UK in an attempt to boost mathematical attainment of lower-achieving pupils in primary schools?
- What are those schemes like, and how effective are they?

The restriction to schemes used and evaluated in the UK is intended to circumvent the potential objection, 'How do we know that it will work here?'.

The aim of this review is to discuss existing interventions and their known results, in order to guide the choice and development of suitable Wave 2 and 3 interventions. The focus is specifically on interventions targeted to children with known mathematical difficulties. Thus, programmes that are designed to improve whole-class performance, such as the Ocean Mathematics Project (Williams, 2008); Cognitive Acceleration through Mathematics Education (Shayer and Adhami, 2007); and the Hamilton Maths programme devised by Ruth Merttens, are excluded from the present review, even when they focus on at-risk groups.

Background and problems to be addressed

1.1 Performance in primary mathematics

Overall, primary school children's performance on national assessments has improved since the introduction of the National Numeracy Strategy in 1999, with the proportion of children reaching level 4 or above at the end of Key Stage 2 increasing from 59% to 77% between 1996 and 2007 (Williams, 2008).

British children's position in international comparisons of mathematics performance varies considerably from time to time; but it has just been revealed that English pupils have performed well in the latest TIMSS study, coming seventh in measures of both primary and secondary mathematics achievement.

1.2 The problem of underachievement

Despite the apparent general improvement in test performance, individual differences in arithmetic are very marked, and there is a significant tail of underachievement, which has so far proved harder to change than overall levels of performance. Cockcroft (1982) reported that an average class of 11-year-olds in England is likely to contain the equivalent of a seven-year range in arithmetical ability. Despite many changes in UK education since then, including the introduction of a standard National Curriculum and a National Numeracy Strategy, almost identical results were obtained by Brown et al (2002). Brown and Millett (2003) suggested that if anything there was an increase in the period from 1998 to 2002 in the gap between highest and lowest achieving pupils.

Since then, the problem of severe underachievement has remained consistent and significant, with about 6% of children each year failing to reach level 3 (the expected level for a seven-year-old) at Key Stage 2 (Gross, 2007; Burr, 2008). The problem of underachievement is particularly great for children from economically disadvantaged backgrounds, defined in terms of eligibility for free school meals (Burr, 2008). However, underachievement affects pupils from all backgrounds.

1.3 Children's difficulties with arithmetic

Many children have difficulties with some or most aspects of arithmetic. It is hard to estimate the proportion who have difficulties, since this depends on the criteria that are used (Mazzocco and Myers, 2003; Desoete et al, 2004). Moreover, as arithmetical thinking involves a wide variety of components, there are many forms and causes of arithmetical difficulty, which may assume different degrees of importance in different tasks and situations. It is likely that at least 15% to 20% of the population have difficulties with certain aspects of arithmetic, which are sufficient to cause significant practical and educational problems for the individual (Bynner and Parsons, 1997 and 2005; Every Child a Chance Trust, 2008). The proportion with severe specific difficulties, which are sometimes describable as dyscalculia, is much lower, though still significant: often estimated at around 6%.

1.4 The concept of developmental dyscalculia

The concept of developmental dyscalculia appears to have been introduced in the 1960s and 1970s (Kosc 1974), though there was some earlier awareness that children could be specifically delayed in mathematics. Dyscalculia was formally recognised as a specific learning disability by the DfES in 2001. It was defined as:

'a condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concept, lack an intuitive grasp of numbers, and have problems learning number facts and procedures. Even if they produce a correct answer, or use a correct method, they may do so mechanically and without confidence.'

There is still much debate as to what dyscalculia is; whether and when the term should be used; and whether it should be seen as a separate disorder or the lower end of a continuum of ability or achievement in mathematics. One problem in interpreting studies and drawing conclusions is that findings about the incidence, specificity, nature and outcomes of mathematical difficulties may vary considerably, depending on whether criteria of discrepancy between mathematical performance and IQ, severity of mathematical weaknesses or persistence of mathematical weaknesses (Mazzocco and Myers, 2002; Desoet et al, 2004) are used. Moreover, some people define dyscalculia in functional terms, as involving specific and severe mathematical difficulties without reference to cause, while others regard it as a brain-based disorder, involving abnormality or underdevelopment of the areas of the brain that deal with number. For example, Butterworth (2005, 2008) considers that dyscalculia involves abnormal or absent function of areas of the parietal lobes of the brain, which are known to be involved in number and arithmetic. As a result, children with dyscalculia may have difficulty with very basic number skills (such as counting) selecting the larger of two numbers and even recognising and distinguishing numerosities of less than four – an ability that has been demonstrated in pre-verbal babies. Although the debates about the existence, nature and specificity of dyscalculia are of great theoretical importance, they may not greatly influence the practical aspects of interventions for children with such difficulties. Gross (2007) has proposed a 'pragmatic stance on the emerging dyscalculia label' whereby:

'educational psychologists might want to adopt the scientifically less interesting but educationally more useful approach of taking dyscalculia by its literal meaning (an inability to calculate). They can then start from the assumption that all children who struggle with numbers of the number system are to some extent dyscalculic'.

1.5 Relationships between mathematical difficulties and other learning difficulties

There have at times been debates with regard to whether mathematical difficulties are merely a manifestation of dyslexia or of other language problems (Miles and Miles, 2004). There is sufficient overlap between dyslexia and mathematical difficulties to have resulted in a number of books and resources for teaching mathematics to children with dyslexia: until very recently perhaps more than for children with specific mathematical disabilities (Chinn and Ashcroft, 2003; Clayton and Barnes, 2003; Henderson, 1998; Kay and Yeo, 2003; Miles and Miles, 2004; Yeo, 2003). Lever's (2003) book is directed specifically at mathematical learning difficulties, but based predominantly on experience of teaching children with dyslexia.

There is a significant overlap between diagnoses of dyslexia and dyscalculia, ranging from 20% to 60% in different studies (Butterworth and Yeo, 2004). However, there is definite evidence that mathematical difficulties are not simply a result of dyslexia, other language difficulties, or generally low IQ, as mathematics and reading difficulties can sometimes dissociate. For example, Gross (2007) points out that in the 2005 Key Stage 2 National Curriculum Assessments, 5.9% of Year 6 pupils were achieving below level 3 in mathematics, 6.3% in English, and only 3.9% in both. This finding indicates that it is quite possible for a child to have severe problems in either mathematics or English without problems in the other. It supports earlier findings from several countries about specific mathematical difficulties, e.g. Lewis, Hitch and Walker

(1994) in England; Gross-Tsur et al (1997) in Israel; and Bzufka et al (2000) in Germany.

There is little evidence at present that children with specific mathematical difficulties are fundamentally different, or need to be taught in different ways, from children whose mathematical difficulties are linked more to other problems. In all cases of mathematical difficulties, children show considerable diversity of strengths and weaknesses within mathematics, and it is arguable that the nature of these specific strengths and weaknesses should be the main determining factor in the types of intervention that they receive.

1.6 Arithmetic is made up of many components

As stated earlier (Dowker, 2004, 2005), there is by now overwhelming evidence that arithmetic is not a single unitary ability at which people are either 'good' or 'bad'. This evidence comes from many converging sources, including experimental, educational and factor analytic studies of typically developing children and adults (e.g. Dowker, 1998, 2005; Ginsburg, 1977; Siegler, 1988); studies of children with mathematical difficulties (Butterworth, 2004; Dowker, 2005; Geary and Hoard, 2005; Russell and Ginsburg, 1984); and functional brain imaging studies (Castelli et al 2006; Dehaene et al 1999; Gruber et al 2001; Rickard et al 2000).

The broad components of arithmetical ability include counting, memory for arithmetical facts, the understanding of concepts, and the ability to follow procedures. Each of these broad components has, in turn, a number of narrower components: for example, counting includes knowledge of the counting sequence, ability to follow counting procedures in counting sets of objects, and understanding of the principles of counting: e.g. that the last number in a count sequence represents the number of objects in the set, and that counting a set of objects in different orders will give the same answer (Greeno, et al 1984; Munn, 1997).

Moreover, though the different components often correlate with one another, weaknesses in any one of them can occur relatively independently of weaknesses in the others. Weakness in even one component can ultimately take its toll on performance in other components, partly because difficulty with one component may increase the risk of the child relying exclusively on another component, and failing to perceive and use relationships between different arithmetical processes and problems; and partly because when children fail at certain tasks, they may come to perceive themselves as 'no good at maths' and develop a negative attitude to the subject. However, the components described here are not seen as a hierarchy. A child may perform well at an apparently difficult task (e.g. word problem solving) while performing poorly at an apparently easier component (e.g. remembering the counting word sequence). Though certain specific components may frequently form the basis for learning other specific components, they need not always be prerequisites. Several studies (e.g. Denvir and Brown, 1986) have suggested that it is not possible to establish a strict hierarchy whereby any one component invariably precedes another component.

The situation may be further complicated by the fact that children with mathematical difficulties can show seemingly random variations in their performance from day to day (Houssart, 2007). This is in fact true of all children (Dowker, 2005; Siegler and Jenkins, 1989) but may increase diagnostic difficulties with regard to children who do have difficulties.

1.7 Some aspects that present difficulty for children

Although children may be identified as having very specific barriers to learning overall, there are aspects of arithmetic that create particular problems for many children. The most consistent finding about children with mathematical difficulties is that many have particular difficulty in retrieving arithmetic facts resulting in an excessive reliance on counting strategies, at an age when other children of the same age are relying much more on fact retrieval (Russell and Ginsburg, 1984; Ostad, 1997, 1998; Cumming and Elkins, 1999; Gifford, 2005). Word problem solving is also often difficult (Russell and Ginsburg, 1984); and so is multi-step arithmetic (Bryant, Bryant and Hammill, 2000).

1.8 Relationship between mathematical difficulties and other areas of cognition

Most researchers do agree that mathematical difficulties are often, though not always, correlated with problems in other areas, and can be linked to language difficulties, spatial difficulties, and/or difficulties with aspects of memory (Chinn, 2004; Dowker, 2004, 2005; Gifford, 2005; Hannell, 2005).

Language difficulties can affect children's ability to understand and make use of instruction, and their ability to encode and represent mathematical information. It also affects their ability to reflect on their own difficulties and work out useful strategies, or ask effectively for help. Hannell (2005, p.7) quotes a secondary school pupil: 'How do I know what it is I don't know? All I know is, I just don't get it.' There are many reasons why children may find it difficult to monitor their own mathematical thinking, and most young children have limitations in doing so, but certainly language difficulties are likely to increase the problem. Moreover, children with even mild language difficulties are likely to have difficulty in rote memory for mathematical information, such as the counting sequence, and, later on, arithmetical facts such as multiplication tables.

Visual-spatial difficulties will have particularly direct impact on such topics as geometry and measurement, but may also affect arithmetic. Severe spatial difficulties may affect children's internal representations of numbers. There is much evidence that most of us represent numbers and their relationships on an internal number line, though it is less clear to what extent this is affected by individual differences in spatial ability within the normal range. Moreover, visual-spatial difficulties may affect children's ability to set out numbers and arithmetic problems in written form, and their ability to use and understand some of the concrete materials used in teaching arithmetic.

Memory difficulties that affect arithmetic can be of two main sorts: problems in storing and/or retrieving arithmetic facts in long-term memory, and problems with working memory: organising and keeping track of items and their order in memory while using these to solve problems. Problems with working memory can have particular impact on mental arithmetic, where children may have trouble in keeping track of their steps in solving an arithmetic problem, and may forget the first step in solving an arithmetic problem by the time they get to later steps. This difficulty may be one reason for the fact that multi-step arithmetic is one of the aspects that children often find difficult. Difficulties in arithmetic are sometimes associated with rather specific memory problems for numbers – in which case it can be hard to tell which is cause and which is effect, but such difficulties are sometimes associated with broader problems in verbal memory, spatial memory or both.

1.9 Principles of intervention

Dowker (2004) set out some general principles for intervention for children with mathematical difficulties. The report recommended that interventions should be individualised; but that in many cases they do not need to be very time-consuming or intensive to be effective. Interventions can take place at any time in a child's school career, but ideally should take place relatively early, both because mathematical difficulties can affect performance in other areas of the curriculum, and in order to reduce the risk of children developing negative attitudes and anxiety about mathematics. Interventions that focus on the specific components with which a particular child has difficulty are likely to be more effective than 'one size fits all' programmes. Therefore, intervention schemes should involve assessments of children's specific strengths and weaknesses within mathematics so that each individual child's weaknesses can be targeted effectively.

1.10 Three Waves of support

The Three Waves

Effective inclusive provision has been summarised in the National Strategies' Waves model which describes a strategic approach to teaching and additional intervention designed to minimise underachievement for all learners. The Waves model can be extended to incorporate additional challenge for all learners, including gifted and talented learners, and can be used as a strategic approach to developing the broader idea of personalisation.

Wave 1 – Quality first teaching

High-quality inclusive teaching is supported by effective whole-school policies and frameworks, clearly targeted at all learners' needs and prior learning. This teaching needs to be based in planning and schemes of work that are designed to move all learners from where they are to where they need to be.

Where there are large numbers of learners who share the same learning needs, the best solution is to adjust the planning to cater for them. It means setting a new trajectory for the learning programme to take learners to where they need to be in terms of age-related expectations. Effective Wave 1 teaching anticipates the needs of learners based on good use of yearly transition data and information.

Wave 2 – Wave 1 plus additional, time-limited, tailored intervention support programmes

Wave 2 provision is designed to increase rates of progress and secure learning for groups of learners that puts them back on course to meet or exceed national expectations. This usually takes the form of a tight, structured programme of small-group support that has an evidence base of impact on progress. This support is carefully targeted according to analysis of need and is delivered by teachers or teaching assistants who have the skills to help learners achieve their learning objectives. The progress of learners is closely tracked for impact. This support can occur outside (but in addition to) whole-class lessons, or be built into mainstream lessons as part of guided work. Critically, intervention support needs to help children and young people apply their learning in mainstream lessons, and to ensure that motivation and progress in learning are sustained. The outcome of Wave 2 intervention is for learners to be back on track to meet or exceed national expectations at the end of the key stage.

Wave 3 – Wave 1 plus increasingly individualised programmes, based on independent evidence of what works

Expectations are to accelerate and maximise progress and to minimise performance gaps. This may involve support from a specialist teacher, highly trained teaching assistant, or academic mentor delivered one-to-one or to very small groups to support learners towards the achievement of very specific targets.

1.11 Supporting children with gaps in their mathematical understanding: the Primary National Strategy Wave 3 mathematics materials

The Primary National Strategy (PNS) Wave 3 materials for mathematics emphasise individualised diagnosis of the errors and misconceptions shown by children with significant difficulties, specific or non-specific, with mathematical learning (usually, children who are performing at least one National Curriculum level below age-related expectations). These materials were piloted in 27 LAs in 2003, and the early stages of their use have been described by Dowker (2004).

The materials include assessment tools to determine conceptual and procedural difficulties in coping with the objectives of the major areas of the mathematics curriculum. A series of booklets have been developed, each of which addresses a particular area of potential difficulty. For example, one of the important skills at Year 2 in the area of Addition and Subtraction is to understand the inverse relationship between the two functions (e.g. if 4 + 13 = 17, then 17 – 4 = 13). Difficulties could involve a complete failure to apply the inverse principle, or its application only to small numbers, for example those below ten. This difficulty can be assessed by giving the child such problems as 'What is the answer to 30 add 20? If 30 add 20 is 50, what is 50 subtract 20?'

The materials were developed for use with children to correct the errors and misconceptions that had been identified through teacher assessment. For example, children who have difficulty in understanding the values of digits may be shown three-digit numbers made out of arrow cards. The teacher then replaces one of the digits (e.g. changing 233 to 203) and asks the child how it has changed. Another related activity involves using a number fan to make a three-digit number (e.g. 523) and asking the child to change one digit to make a larger number; such activities can also be carried out with two-digit, four-digit or higher numbers.

Many of the materials and activities involve multisensory and multi-context learning (see 1.13). Children are presented with a variety of words, symbols, models and images to represent the same concept or process. For example, activities for children who have difficulty with remembering and using multiplication facts include representing such facts by groups of cubes, beads and domino patterns; verbal counting by twos or larger sets; and representing the facts on a multiplication grid.

The materials are designed for use within or outside the daily mathematics lesson, and may be used individually by the child with the class teacher, a teaching assistant, or a special needs teacher. Though the exact mode of teaching is up to the school, each child typically undergoes one 20-minute individual session per week, and five-minute 'spotlight' sessions on each of the following days, in addition to following the mathematics curriculum with or without specialist support.

As Haseler (2008) points out, such interventions have the advantage, not only of being more individualised and intensive, but of being more specifically targeted to individual children's difficulties.

> *'An additional difficulty with 'ready made' programmes is that they dictate how each topic is taught, regardless of the child's particular difficulties…. This information is important because we may need to adapt our teaching style or adopt a different approach if we are to help that particular pupil overcome their difficulties. There is greater flexibility with Wave 3 materials, which includes some suggestions on what to do if a pupil gives an incorrect response.'*

Evaluation of the effectiveness of the Primary National Strategy Wave 3 materials is still in a relatively early stage. Gross (2007) has reported feedback from LAs on the pilot study in 2003-2005, and on the use of Wave 3 materials at national level since 2005.

The feedback on the pilot work was on the whole positive. Most staff and pupils enjoyed the materials and 'schools reported that the pilot had helped them move on from a sole focus on literacy in their special needs support' (Gross, 2007). However, some schools found it difficult to implement the detailed assessment, due to lack of training and resources, and just used the activities with less able pupils when related topics came up in the course of the curriculum. In the cases where the materials were used most successfully, for example in Bromley, the LA provided training by specialist teachers for senior staff members, classroom teachers and teaching assistants, and gradually handed more and more of the intervention support to the teaching assistants, once they were more familiar with the materials and procedures. This increased confidence among the teaching assistants themselves, and resulted in more effective, targeted and sustained use of the intervention materials in the schools.

Once the materials were introduced at national level, evaluations continued so far to be mostly of a qualitative nature. Schools reported that the children involved in the Wave 3 interventions showed more positive attitudes, greater self-esteem, and greater willingness to take part in mathematical activities. Some LAs undertook their own more quantitative evaluations, usually carried out by educational psychologists. For example, children in 14 schools in Norfolk were tested with the BEAM diagnostic interviews in number sense (Denvir and Bibby, 2001) at the beginning and end of a six-month intervention period using the Wave 3 materials. Their performance in National Curriculum tests was also recorded. Almost all the pupils made greater progress during the intervention period than in previous equivalent periods, and many had increased their National Curriculum levels by at least one sublevel. The greatest gains were in schools that used the materials systematically and with effective support from senior staff.

1.12 Materials that represent numbers and operations as part of Wave 3 intervention

Some programmes are based on the view that many children with mathematical difficulties have failed to build up adequate representations – verbal, visual and abstract – of numbers and arithmetical operations (Haseler, 2008). For example, they may see each number just as a collection of ones, which impedes the efficiency of their manipulations of numbers, and affects the depth of their understanding. A number of types of apparatus have been developed to help children build up images of number. Often such programmes emphasise not just use of apparatus but the use of multisensory teaching, to strengthen representations of number and also to enable children who find it difficult to form images in one modality (e.g. visual) to do so in another (e.g. tactile).

There are many types of apparatus that are frequently used in primary mathematics teaching: e.g. counters, dice, dominoes, linking cubes and base ten sticks. There are also some that are used in a more specialised way in interventions: e.g. the ten frames used in Mathematics Recovery, and Numicon which forms the basis of one of the interventions currently used in several LAs.

1.13 Recommendations of the Williams Report: Independent Review of Mathematics Teaching in Early Years Settings and Primary Schools

Williams (2008) has put forward a strong recommendation for early intervention for primary school children who are experiencing difficulties in mathematics (Recommendation 8 of his review). In particular, he recommends that children with serious difficulties in mathematics should receive intensive one-to-one intervention from a qualified teacher, though paired or small group work may be appropriate in some instances.

Children with somewhat less severe difficulties might receive less intensive individualised or small-group intervention, and teaching assistants could provide some of this, with appropriate training. Typically, intervention should last for approximately 12 weeks, or one term. Williams proposes that mathematics interventions should be given in Key Stage 1, preferably in Year 2. Effects of interventions should be studied, with the establishment and use of appropriate diagnostic methods to assess performance at the time of intervention and on completing the intervention.

Less intensive Wave 3 and Wave 2 interventions could be led by appropriately trained teaching assistants; consideration should be given to the training required and the use of interventions, with a robust evidence base of impact on learning and progress.

Williams also recommends that a longitudinal study should be commissioned to assess the long-term benefits of intervention both at Key Stage 2 and, eventually, at GCSE level.

1.14 Every Child Counts

In 2008, the Every Child Counts programme was established to both inform and respond to the recommendations of the Williams review by developing an intensive, teacher-led Wave 3 intervention for the lowest-achieving children in Year 2, as well as to enable all primary school children with mathematical difficulties to receive effective support through the three waves of intervention model.

Every Child Counts is a partnership initiative between the Every Child a Chance charity (a coalition of business partners and charitable trusts), and government (DCSF and the National Strategies).

The aim is to enable the lowest-attaining children to make greater progress towards expected levels of attainment in mathematics, catching up with their peers and achieving level 2 or better and, wherever possible, level 2b or better by the end of Key Stage 1. The intention is to provide, from the academic year 2010/11, intensive support in mathematics to 30,000 Year 2 children annually. It is also expected that the highly trained teachers providing this support will have a wider impact on learning and teaching in their schools, and help to raise standards across the board.

Every Child Counts has three phases:

- research (academic year 2007/08);
- development (2008/09 and 2009/10);
- national roll-out (2010/11 onwards).

Research phase (2007/08)

The first phase of the initiative (2007/08) ran alongside, informed and was informed by the Williams review of the teaching of primary mathematics. Between September and December 2007 information was gathered through visits to different intervention schemes that were currently in place in LAs. This provided a baseline for further activity by summarising:

- the programmes/approaches in use;
- their training and support infrastructures;
- evidence of their impact on children's learning and progress.

Between January and July 2008, three existing models of intervention support (Mathematics Recovery, Multisensory Mathematics (developed in Leeds LA) and Numeracy Recovery (developed in Hackney) were extended to new LA areas in order to identify the impact and draw out the essential features the national programme should incorporate to ensure success.

Five LAs were involved. Ten schools were involved in each area. Support included funding for a dedicated LA lead consultant and dedicated teachers in the participating schools; expert training by professionals experienced with the specific interventions; and follow-up guidance from the trainers, the PNS and the Every Child a Chance Trust. Outcomes are summarised in Chapter 3 of this report.

Chapter 2: Method

LA advisers responsible for primary mathematics were contacted and asked for information about the intervention schemes that they had developed and/or were using for children experiencing difficulties. Simultaneously, the devisers of other known initiatives were contacted: for example, those involved in devising and establishing Numeracy Recovery, Mathematics Recovery, Catch Up Numeracy and Talking Maths. Recently published literature on mathematics interventions was also consulted, especially material published since 2004.

2.1 Information sought

Overview of scheme *(origin, aim, development, intended outcomes)*
Target pupils *(age group(s), criteria for selection of children, entry and exit descriptors)*
Delivery details *(teaching assistant/teacher, trained volunteers, duration, number of lessons per week and length of session, one-to-one/small group, size of group (if applicable))*
Training/Professional development details
Description of scheme *(pedagogical principles, activities, any links to Primary Framework for mathematics, links to published materials)*
Evaluation details (complete sections as applicable) **Main reference (e.g. source, author, publication details)** **Research design** **Age range** **Type of children** **Number of experimental group** **Number of comparison group** **Number of alternative treatment group** **Length of intervention evaluation period (in weeks)** **Test(s) used** **Standardised** **Other** **Follow-up studies** **Summary of data submitted for this research review (please attach data to this form)**
Any other relevant information you wish to submit

2.2 Analysis

The material was collated, reviewed and, where possible, analysed. The original aim was to analyse all the material within a common framework, including (1) ratio gains – the amount of progress in mathematics, in months, divided by the time in months during which the gains were made; and (2) effect sizes – the difference in the amount of progress made by the children in the initiative and children in a control group, divided by the standard deviation of the control group's pre-test average score.

In fact, though many of the schemes had undergone some form of evaluation, most are still in an early stage; the evaluations used very diverse measures; and most did not include ratio gains or effect sizes or data from which these could be obtained. Therefore, the review and summary of the data are more qualitative at this stage than originally intended.

Chapter 3: Conclusions and recommendations

3.1 Main findings

3.1.1 Interventions are increasingly widely used

Compared with the situation a few years ago (Dowker, 2004) it appears that, largely as a result of government recommendations and financial support, and in particular the introduction of the PNS Wave 3 materials 'Supporting children with gaps in their mathematical understanding', there has been a great recent increase in the use of individualised interventions.

3.1.2 Interventions are varied, but most come into a few broad categories

The interventions are very varied, but most of those submitted to the report come into three main categories:

i. those that involve use of the existing PNS Wave 3 materials, usually with some modifications;

ii. those that are based primarily on detailed diagnostic assessments of individual strengths and weaknesses, with activities targeted to these;

iii. those that primarily involve the use of multisensory apparatus such as Numicon.

There are overlaps between these categories, and there are some interventions that do not clearly fit into any of these categories, e.g. Lambeth's use of the JUMP mathematics programme.

3.1.3 Interventions are viewed positively by teachers and LAs

Those schools and LAs that chose to report on intervention schemes report positive views of them. They are described as improving performance and attitudes in children, and it is often stated that teachers and pupils enjoy them. Positive influences on the professional development of teachers and teaching assistants are also reported. It is sometimes reported that the schools improved their overall performance in National Curriculum tests.

3.1.4 Test scores indicate positive effects on performance.

When performance before and after intervention has been tested, children have generally shown improvements in their performance. Though standardised tests were sometimes used as a measure, the commonest measure was performance in National Curriculum tests. In most studies, children made significant progress, with a large number progressing by at least two National Curriculum sublevels in a year, which is the expected level of progress within the time for all children, and a considerable acceleration on the previous performance of the low-attaining children.

3.1.5 Reports indicate that the success of an intervention scheme depends significantly on effective management and training

It is not sufficient to simply provide materials and activities and expect that these in themselves will result in improved performance. There needs to be guidance and training for teachers and teaching assistants as to how to provide the assessments and the interventions. There also has to be support within the schools for the interventions, including ensuring that the time is in fact available for the interventions to be carried out.

3.2. Problems in interpreting scores and comparing levels of improvement

We are still, however, at an early stage, probably a much earlier stage for mathematics than for literacy, as regards our ability to compare levels of improvement with different mathematics intervention schemes. Problems that make such evaluations difficult include:

3.2.1 Possible exclusion of information about unsuccessful schemes

If a school or LA finds a scheme ineffective or disappointing it may give up using it at an early stage, and in any case is less likely to prepare and submit a report to an investigation such as the present one. This means that less information is available about schemes that proved disappointing, and thus about the pitfalls to be avoided.

3.2.2 Not all schemes have yet been assessed

Some schemes that are described in this report have yet to be assessed. Some others are still in an early stage of being assessed.

3.2.3 Not all statistical information is available

The evaluations were generally carried out by schools and LAs, whose primary business is education, not research; and calculations of ratio gains and effect sizes were not usually carried out. Statistical levels of significance were not always assessed.

3.2.4 Most studies did not have control groups

Children's levels of performance were usually compared with their own earlier performance, or with general levels of progress and performance expected of children starting at a similar point. Control groups were usually not available. This means that the possible effects of other changes with time were not taken into consideration. It also means that the general effect of the school's being part of a special programme ('Hawthorn effect') was not taken into account. In the case of one programme, Catch Up Numeracy, where controls in the same schools were studied, it was found that they did make more progress than would normally be expected of low-attaining children, though significantly less than that of the children in the intervention programme. The use of controls in a wider range of studies would make it easier to assess the specific effects of involvement in the programme.

3.2.5 Lack of comparison between different interventions

It was generally not possible to compare the effects of different interventions, and to see whether particular interventions are better for all children or for certain groups of children; or to assess the general influence of individualised intervention as such, as distinct from the specific effects of a given programme.

The Every Child Counts Research Phase study was aimed at filling part of this gap, by securing a comparison study between three different intensive interventions.

3.3. Every Child Counts Research Phase study

This study examined and compared performance by children in five LAs, using three intervention programmes: Mathematics Recovery in two LAs, Numeracy Recovery in two LAs, and Numicon in one LA (see Chapter 3 for findings with regard to specific interventions). Its most recent conclusions were:

3.3.1 Children made gains in standardised scores on an NFER test and in National Curriculum sublevels

When tested at exit, only 14% of children were working at or below the threshold level of the test and had a standardised score set at the floor of the test, and almost half the children (49%) had a standardised score of 80 points or more.

Over the period of intensive support, the average gain made by the children in the standardised scores was just under ten points, and the average gain in sublevels was 1.86 sublevels as measured by the standardised test and 1.22 sublevels as measured by teacher assessment.

Children made gains when assessed by teachers using National Curriculum sublevels. At exit, teachers assessed nearly a third of children (29%) to be working at low or secure level 2 and over a quarter of children (27%) to be a high level 1 (level 1a).

3.3.2 Lack of evidence for effect of length of intervention

There is little evidence at this stage of a correlation between the length of the intensive support programme children receive and the gains made. The intensive support programmes were curtailed for the purposes of the research.

3.3.3 No clear evidence at this stage for any of the programmes being more effective than others

It is too early to compare gains and to attribute the improvements made to any particular features of the different models of support. There were gains made in each LA. It will be necessary to examine the effects of background variables in making any later comparison.

3.4 Recommendations

3.4.1 Individually targeted interventions appear worthwhile and should be continued and further developed

The last few years have brought greatly increasing use of individually targeted interventions for children with mathematical difficulties. These interventions appear to give positive results and to be regarded favourably by teachers and pupils. It is strongly recommended that schools should continue to use and develop such programmes as they develop the layered approach to intervention of the Every Child Counts programme. Though there have been few systematically controlled studies (see 4.4.2 and 4.4.7), the findings so far appear to support the view that any extra help in arithmetic is likely to give some benefit; however, interventions that focus on the particular components with which an individual child has difficulty are likely to be more effective than those which assume that all children's arithmetical difficulties are similar.

3.4.2 So far, there is no evidence that any one programme is best for most or all children

At present, there is no evidence for any one programme being the best. This is partly due to the limitations so far in evaluations, as described in section 4.2. Far more systematic research is needed in order to compare different programmes.

Even when such research is done, it is likely that there will prove to be no single best intervention programme, and that different programmes would be suitable for different groups of children. For example, highly intensive programmes such as Mathematics Recovery and Numeracy Recovery might be more suited to children with severe difficulties; less intensive programmes such as Catch Up Numeracy might be more suited to children with milder difficulties. The Williams Review (2008) drew particular attention to the features of Numeracy Recovery and Mathematics Recovery, both of which have demonstrated considerable success in enabling low-attaining children to make progress, and in keeping with the recommendations of this review, the development of the new national Numbers Count intervention has been informed by these features. For further information about Numbers Count, see Appendix 1 on p44.

Until there is more research evidence as to which programmes are best for which groups, it is recommended that teachers, numeracy consultants and others exercise their judgement about which programmes appear most suited to the children with whom they are working, with the prime point of reference being the evidence base that the approach is impacting on attainment.

One question that arises in this context is whether small group intervention can be as effective as individualised intervention. There are controversies over this issue (Williams, 2008). The preliminary evidence of the Every Child Counts Research Phase suggests that for children who have significant weaknesses individualised intervention is better, both because it can be more precisely targeted and because such children have often developed educationally maladaptive strategies to cope in group situations, i.e. strategies of hiding their difficulties, which may make them harder to diagnose and overcome. Small-group teaching may, however, provide useful opportunities for discussion and reflection about strategies, and for 'peer tuition'; and it may be beneficial for children with very mild difficulties and/or as a supplement or follow-up treatment for children who also undergo individualised intervention. This area will be the focus of a robust external evaluation as part of the development of Numbers Count.

3.4.3 Importance of appropriate management, guidance and training

As indicated in section 4.1 and in other research reports (e.g. Gross, 2007) and educational recommendations (Hannington, 2008; Haseler, 2008; Thomas and Allingham, 2008), it is important that programmes be managed appropriately by school leadership, both in terms of providing adequate guidance, training and support for staff involved in intervention work, and in terms of ensuring that adequate time and resources are available.

3.4.4 Importance of suitable assessments

Effective interventions imply some form of assessment, whether formal or informal, to (a) indicate the strengths, weaknesses and educational needs of an individual or group; and (b) to evaluate the effectiveness of the intervention in improving performance.

If assessment is over-emphasised, especially if the assessment involves only one or two aspects of mathematics, there is a risk of over-simplification of pupils' problems, and of teachers and teaching assistants feeling pressured to 'teach to the test'. However, if there is no assessment, or no adequate guidance as to carrying out assessments, then it is much more difficult to carry out targeted interventions. One barrier to the effective use of the PNS Wave 3 materials 'Supporting children with gaps in their mathematical understanding' is that there has often been insufficient provision of guidance and time for assessment (Gross, 2007). Explicit provision of such guidance and resources improves the effectiveness of such interventions (Gross, 2007; also see section 3.1 of this report). The use of effective assessment, both standardised and diagnostic, is a key aspect of the development of the Numbers Count programme.

Butterworth (2003) has devised a computerised screening test of basic numerical skills: incorporating the recognition of small numerosities; estimation of somewhat larger numerosities; and comparisons of number size. These are particularly intended to identify severe arithmetical difficulties (dyscalculia) rather than milder difficulties.

National Curriculum tests, and standardised tests, such as the British Abilities Scales Number Skills Test and the Basic Number Screening Test, can be useful in deciding who should benefit from an intervention programme and in monitoring the effectiveness of such interventions. A disadvantage is that most such tests give a global measure of mathematical ability and do not look at different components separately.

There are an increasing number of tests (mostly not as yet fully standardised) that do assess different components of mathematics separately: for example, the diagnostic assessments used in the schemes described in section 3:2 of this report; also see Lawson (2008).

A combination of standardised testing and more informal diagnostic interviews may give particularly useful information. It may often be impractical for teachers and teaching assistants to carry out such detailed and varied assessments, but Mackenzie (2007) has strongly recommended them for educational psychologists.

3.4.5 Appropriate use, development and testing of apparatus and multisensory and multi-context teaching

There is evidence that use of structured apparatus and multisensory teaching can be helpful and effective in interventions, especially with children who have language difficulties or who have failed to acquire useful representations of number.

Learning tends to be more effective if it occurs in several different contexts and children are encouraged to relate them to each other. Otherwise, children may learn very efficiently in one context but not apply it to others. At one time, it was thought that if children are initially encouraged to use concrete objects to solve arithmetic problems, they will later be able to transfer their learning to other, more abstract problems. However, it has been found that children do not always make this connection, and even if they eventually learn how to do a particular type of problem in both concrete and numerical form, they may not connect the two. As a child informed Hart (1989), 'Bricks is bricks and sums is sums'. Moreover, children may not always understand the concrete materials themselves, or how these are related to number (Dowker, 2005; Hannell, 2005). Hannell (2005, p. 28) quotes an adult reminiscing about his school experiences:

> *I never did really get what those little wooden blocks were all about. I said they were 'tens' and 'ones' because that was what the teacher said we had to call them; but it never, ever dawned on me that they could stand for anything real like ten kids, or ten dollars. They were just little bits of wood that we did things with.*

Hence, it is important to present materials in a variety of contexts and a variety of sensory modalities, and encourage children to make links (Fuson, 1992). The use of a multisensory approach, and of materials that lend themselves well to such an approach, are currently recommended by numerous educators (Haseler, 2008; Henderson, Carne and Brough, 2003; Thomas and Allingham, 2008) and advocated in the Williams Review (Williams, 2008). At the same time, it is pointed out that simply giving children the materials may not be sufficient; as Haseler (p.240) states:

> *It is also important to remember that however 'good' we believe the equipment to be, it will only be of value to pupils if it is used by suitably trained staff who understand the rationale behind it.*

It is important to investigate the best methods of using such equipment, and also the extent to which it may be more suited to some children than others (Dowker, 2005). It is also important to investigate whether there is any difference in effectiveness between different types of multisensory apparatus. Numicon is the most often reported, but there are known to be other schemes such as Addacus (also mentioned in section 3.3.2).

3.4.6 Consideration of ways in which ICT may be used in interventions

Given the ubiquity and popularity of computers and computer games, it would be worth investigating whether and how computerised games and activities might be used in interventions. The evidence so far has been that computerised interventions are less effective than some other interventions (Kroesbergen and Van Luit, 2003). However, there are two characteristics of much computerised intervention in the past that may contribute to such findings: they have tended to be used as a substitute for personal attention from teachers and others, and they have tended to emphasise factual and procedural learning rather than conceptual learning. There is room for investigation of the uses of computerised interventions which would include conceptual as well as factual and procedural learning, and which might supplement rather than replace teacher involvement. As yet, research in this area is at a very early stage (Al-Musawi, 2007; Wilson and Dehaene, 2007) and there is much room for further work in the area.

3.4.7 Further research and evaluation should be carried out

As suggested throughout the report, and as recommended by Williams (2008), it is important to carry out further research comparing the effectiveness of different interventions, and comparing all of them with similar amounts of individual attention. It is important to investigate the relative effectiveness of interventions for children with different characteristics: e.g. age on entering programmes; level and type of arithmetical difficulties; and performance on other cognitive tests such as verbal and spatial ability.

3.4.8 Different types of mathematics

This report has treated the terms ‘mathematical ability’ and ‘arithmetical ability’ as virtually identical, because most intervention studies so far have looked predominantly at arithmetic. It would be desirable to develop and investigate interventions involving other areas of mathematics, such as measurement, shape and space, and data handling.

3.4.9 Extending the age range for the study of interventions

Although it appears ideal for interventions to be carried out in the early primary years, some children for various reasons may not receive such interventions, or may not benefit maximally from them. Also, some mathematical difficulties may only show themselves at a later stage. It would therefore be desirable to carry out more studies of the effectiveness of arithmetical interventions with older children and adults.

At the other end of the scale, it would be desirable to study the extent to which specific strengths and weaknesses in counting and other early numerical concepts and skills in the Foundation Stage may predict general and specific strengths and weaknesses later on. If they do, then it may be possible to adapt intervention techniques to ameliorate mathematical difficulties very early on, with a view to reducing or even preventing subsequent problems.

Chapter 4: Schemes used in primary mathematics intervention

The mathematics interventions reviewed are very diverse, but can be placed in four broad categories:

- those that predominantly involve use of the existing PNS Wave 3 materials 'Supporting children with gaps in their mathematical understanding';
- those that involve componential theories of arithmetic, with interventions targeted to individual strengths and weaknesses following detailed assessments;
- those that focus on the use of structured mathematical apparatus;
- those not falling into any of the above groups.

4.1 Interventions focusing on the Primary National Strategy Wave 3 materials 'Supporting children with gaps in their mathematical understanding' and other existing government-sponsored materials

4.1:1 Snap on 2 Maths in Surrey

Snap on 2 Maths is designed for children in Key Stages 1 and 2 who are working significantly below age-related expectations. It was originally aimed at bringing children in Key Stage 1 up to level 2c, but has since been extended to helping children who are at level 2c at the beginning of Key Stage 2 to reach level 4 by the end of Key Stage 2.

The programme involves teaching assistants delivering a daily ten- to 15-minute lesson to a small group of approximately four children. Children are given a diagnostic assessment, in order to establish their level of mathematics performance, with respect to the National Curriculum, at the beginning of the programme. They are given a detailed programme of intervention involving activities and resources from the PNS Wave 3 materials and the 'Models and images' charts from the Primary Framework. Children undergo regular reviews to check their progress.

A total of 27 schools were involved in the project and from each school a Year 3 teacher, a mathematics subject leader and a teaching assistant underwent training for the programme.

Evaluation

162 children, comprising 90 boys and 72 girls, were followed from Year 2 to Year 3. 50% moved forward by at least two National Curriculum sublevels within one year (an acceleration of their previous performance) and 11% moved forward by more than that amount. Teachers in the schools expressed favourable views of the programme.

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4.1:2 West Sussex Wave 3 Mathematics Project

This programme was devised as a potential means of making best use of PNS Wave 3 materials to improve mathematical performance; in West Sussex schools. Schools received a pack of the PNS Wave 3 materials, containing two sets of booklets focusing on common errors and misconceptions in calculation; a booklet listing resources and games; a booklet of advice for teachers on managing the programme; and an interactive CD-ROM providing direct access from tracking charts of pupils' learning to computerised teaching materials.

The LA provided two training sessions: one for mathematics coordinators and special educational needs coordinators (SENCOs) and one for teaching assistants. Ten schools took part in the project in 2005/06, and another eight in 2006/07.

Most schools targeted children in Year 4, but Year 3, 5 and 8 pupils were the focus in some schools. Some schools targeted the lowest-ability pupils, and some the next level up. The most usual ways of identifying children to benefit from the programme were teacher judgement and results from national and QCA assessment tests. The tracking charts were not used to a great extent.

Evaluation

Two schools reported their results in 2005/2006. In one school, 14 out of 16 pupils showed improvement. In the other, the four who started at level 2c made an average of 1.75 sublevels of progress, while the six who started at level 2b made only limited progress. In 2006/07, one school reported that of five Year 6 pupils in the project, three moved from level 2 to level 3b or 3c, one from level 3c to 3b, and one remained at level 3c. Another school reported that of three Year 5 pupils who started in September at level 1a, two had reached level 2a by March, and one had reached level 3c.

The schools were happy with the project overall. They emphasised the need to make sure that all necessary resources were present, and to have ways of measuring impact.

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4.1:3 Moving on in Maths, Luton

This programme was based on a model in Peterborough LA, for Year 6 pupils who are performing somewhat below expected level, and are seen as, with the help of additional intervention support, having the potential to achieve level 4 in National Curriculum assessments. Such pupils were identified on the basis of their performance on practice National Curriculum tests. The pupils involved in the programme were monitored closely and received support through available focused interventions, including Springboard 5 and 6, published materials such as BEAM Maths, and an in-school created '100 Maths Facts PowerPoint'. Twelve schools were used in the pilot project, and the number of schools involved increased to 27 in the following year.

Evaluation

Pupils' Key Stage 2 results were used for the evaluation of the pilot project. Percentages of pupils reaching level 4 in the 12 relevant schools were compared for 2004 and 2005 (before the programme began) and again in 2006. Scores increased considerably in most of the schools, suggesting a positive impact. The schools also showed significant gains in the numbers reaching level 5. Teachers viewed the project positively.

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4.1:4 Making Maths Magic

This is an intervention developed in the East Riding of Yorkshire. It involves work in small groups, usually including no more than six children. Interventions are given by teaching assistants after one day's training, and include four 30-minute sessions per week over a six-week period. It can be implemented either in the summer term of Year 1 or the autumn term of Year 2. Approximately six children in Year 1 or 2 from each of 12 schools took part in the study.

Each intervention includes a five-to-ten-minute starter activity (e.g. counting; number-related songs; memory games); a 20-minute main activity dealing with the mathematics objective being addressed; and a five-to-ten-minute reflection activity, where the children are asked questions to assess their understanding, and to encourage them to reflect on the topic.

The activities involve use of the PNS Wave 3 materials to address difficulties in Year 1 mathematics curriculum topics, e.g. within Shape: 'to recognise, name and sort 2D and 3D shapes and use correct language to describe some of their properties'; within Addition and Subtraction: 'to recognise, name and sort 2D and 3D shapes and use correct language to describe some of their properties' and within Measure: 'to use language – longer, shorter; to compare a range of lengths; to measure using non-standard and standard measures (cm).'

Evaluation

The evaluation looked at the total level of performance of all the 426 children in the relevant age group in the 12 schools that took part in the pilot study during the spring term of 2006 (rather than the specific children involved in the intervention). The average percentage of pupils achieving at least level 2b at the end of Key Stage 1 was 73.5% in the schools involved in the project, as compared with 77.3% for all pupils in the LA. The average increase from 2005 to 2006 in pupils achieving at least level 2b at the end of Key Stage 1 was 13% in the schools involved in the project, as compared with an average increase of 0.6% for all pupils in the LA.

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4.2 Targeted individualised interventions involving detailed assessments of strengths and weaknesses

4.2.1 Numeracy Recovery developed in Hackney

(Not to be confused with the Numeracy Recovery programme which served as the pilot study for Catch Up Maths, 3.2:4)

This is a relatively intensive teaching programme, delivered to Year 2 pupils who are seriously underachieving in mathematics. It includes a daily one-to-one instruction session, delivered by a specifically trained teacher. This takes place over a period of one term (12 weeks).

Children were selected for the programme on the basis of teacher judgements and the National Foundation for Educational Research (NFER) 6 test at the end of Year 1. Most pupils selected for the programme were at level 1c or below. Children were given an initial diagnostic assessment, with the aim of tailoring the intervention to each individual child with regard to (i) specific strengths and weaknesses with various types of mathematics content; (ii) the strategies and activities likely to motivate the child; and (iii) learning styles.

Parents/carers were involved as much as possible: they met the trainers at the outset to discuss the programme, and were encouraged to come to Numeracy Recovery training sessions on a number of visits. Children were given homework games and activities, and the parents were given advice on how to support children with these.

The specialist teachers liaised with the children's classroom teachers and other relevant staff in organising the assessments and interventions. At the end of the programme, they were given detailed exit assessments that provided specific guidance for the class teacher.

Evaluation

Groups of children in the programme were studied over the three years 2004-2007. Results were measured in terms of changes in National Curriculum sublevels from the end of Year 1 to the end of Key Stage 1 National Curriculum test in Year 2. The total sample size was 412 pupils, with approximately equal numbers in each cohort.

The average gains by year were 2.3 sublevels in 2004-2005; 2.94 in 2005-2006; and 3.15 in 2006-2007.

In 2006-2007, age-standardised scores were also recorded, on the NFER test at the end of Year 1 and the National Curriculum test at the end of Year 2. The average standardised score increased from 78.5 to 93.8, indicating an average gain of 15.3 points.

4.2.1:1 Numeracy Recovery within Every Child Counts Research Phase study

Numeracy Recovery was one of the interventions trialled for seven-year-olds with mathematical difficulties in the research carried out by Every Child Counts, during the summer term of 2008. Like the other interventions in the study, it was supported through funding for a dedicated LA lead consultant and dedicated teachers in participating schools; expert training by professionals experienced with the specific interventions; and follow-up consultancy and support from the trainers, the PNS and Every Child a Chance Trust.

It was studied in ten schools in each of two LAs named 'C' and 'D'. The children were low attainers in Year 2, starting at an average National Curriculum level of just over 1c. There were 39 children in LA C and 43 children in LA D. Results were assessed in terms of gains in National Curriculum points, with 0 representing 'Working toward level 1' and seven representing level 3c; and in terms of standardised scores on the NFER standardised mathematics test.

In LA C, children progressed from an average score of 1.63 National Curriculum points at entry to 3.38 National Curriculum points at exit, giving a mean gain of 1.75 sublevels, corresponding to a move from level 1c to level 1a. By the end of Year 2, 52% had reached at least level 2c, and 27% had reached at least level 2b. On the NFER standardised test, they had progressed from a mean standard score of 74.60 to a mean standard score of 83.05: a mean gain of 8.45 points.

In LA D children progressed from an average score of 1.98 to 4.26 National Curriculum points, giving a mean gain of 2.29 sublevels, representing an average move from level 1c to level 1a. By the end of Year 2, 83% had reached at least level 2c, and 49% had reached at least level 2b. On the NFER standardised test, they progressed from a mean standard score of 76.43 to a mean standard score of 90.83; a mean gain of 14.40 points.

Thus, children in both LAs made significant gains. The somewhat greater improvement in LA D, especially on the standardised test, may be due to the fact that a higher proportion of children in LA D received all their intervention sessions.

4.2.2 Mathematics Recovery

The Mathematics Recovery programme was designed in Australia by Wright and his colleagues (Wright et al, 2000, 2002). The basic principles of the programme have been described by Dowker (2004) and will be set out again here. In this programme, teachers provide intensive individualised intervention to low-attaining pupils: classically, six- and seven-year-olds, though it has also been applied to children in other age groups. Children in the programme undergo 30 minutes of individualised instruction four days a week over a period of up to 12 weeks. The programme is also suitable for small-group tuition.

The choice of topics within the programme is based on the Learning Framework in Number, devised by the researchers. This divides the learning of arithmetic into six broad stages of early arithmetical learning:

- emergent (some simple counting, but few numerical skills);
- perceptual (can count objects and sometimes add small sets of objects that are present);
- figurative (can count well and use 'counting all' strategies to add);
- counting on (can add by 'counting on from the larger number' and subtract by counting down; may have developed counting-up from and counting-down-to strategies; can read numerals up to 100 but little understanding of place value);
- intermediate number (possesses a range of counting strategies including count-down-to for solving addition and subtraction, and can choose the most efficient to solve a problem);
- facile (can use a range of strategies which do not depend on counting-by-ones, e.g. compensation, using a known result, adding to ten, commutativity, subtraction as the inverse of addition, split and jump strategies, and can multiply and divide by strategies based on repeated addition).

Children are assessed, before and after intervention, in a number of key topics. They undergo interventions based on their initial performance in each of the key topics. The key topics that are selected vary with the child's overall stage. For example, the key topics at the emergent stage are:

iv. number word sequences from 1 to 20;

v. numerals from 1 to 10;

vi. counting visible items (objects);

vii. spatial patterns (e.g. counting and recognising dots arranged in domino patterns and in random arrays);

viii. finger patterns (recognising and demonstrating quantities up to five shown by number of fingers);

ix. temporal patterns (counting sounds or movements that take place in a sequence).

The key topics at the next, perceptual, stage are:

i. number word sequences from 1 to 30;

ii. numerals from 1 to 20;

iii. figurative counting (counting on and counting back, where some objects are visible but others are screened);

iv. spatial patterns (more sophisticated use of domino patterns; grouping sets of dots into 'lots of two'; 'lots of four', etc.);

v. finger patterns (recognising, demonstrating and manipulating patterns up to ten shown by numbers of fingers);

vi. equal groups and sharing (identifying equal groups, and partitioning sets into equal groups).

The key topics at later stages place greater emphasis on arithmetic and less on counting. Despite the overall division into stages, the programme acknowledges and adapts to the fact that some children can be at a later stage for some topics than for others.

There are many activities that are used for different topics and stages within the Mathematics Recovery programme. For example, activities dealing with temporal patterns at the emergent stage include children counting the number of chopping movements made with the adult's hands; making chopping movements with his/her hands; producing a requested number of chopping movements with their own hands; counting the number of times they hear the adult clap; and clapping their own hands a requested number of times. Activities dealing with number word sequences in fives at the counting on stage include children being presented with sets of five dot cards; counting the dots as each new card is presented; counting to 30 in fives without counting the dots; counting to 30 in fives without the cards; counting to 50 in fives without the cards; and counting backward in fives from 30, first with and then without the cards.

Mathematics Recovery was introduced to the UK in 1996, initially in north west England but by 2008 it has now been taken up by over 30 LAs in England, Scotland and Wales. In Ireland it has become a national training programme for 312 special education teachers. In 2008 further development took place in Canada, Spain and Mexico.

The key features of Mathematics Recovery, in the UK as in Australia, can be summed up under four headings: Early intervention, Assessment, Teaching and Professional development. Early intervention is given to children when they are falling behind in Year 1, as a means of preventing the gap between their knowledge and that of more able pupils growing too wide and causing them to experience excessive failure. The assessment tools are videotaped diagnostic interviews rather than pencil-and-paper tests and are used to plan the instructional activities. Mathematics Recovery training is also seen as a part of the CPD of teachers, especially those who are not mathematics subject specialists.

Despite the emphasis on early intervention for young children, assessment and teaching materials and activities can be and are used for children up to 12 years old. There are no hard and fast criteria for selection or for exit from the programme.

- *Early Numeracy: Assessment for Teaching and Intervention*, 2nd edition, (2005) Wright, R.J., Martland, J.R. and Stafford, A.K., Sage Publications, London
- *Teaching number: Advancing Children's Skills and Strategies*, 2nd edition, (2006) Wright, R.J., Martland, J.R., Stafford, A.K. and Stanger, G., Sage Publications, London
- *Teaching Number in the Classroom with 4–8 year-olds,* (2006) Wright, R.J., Stanger, G, Stafford, A.K. and Martland, J.R., Sage Publications, London

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4.2.2:1 Evaluation of Mathematics Recovery within Every Child Counts Research Phase Study

Mathematics Recovery was one of the interventions trialled for seven-year-olds with mathematical difficulties in the research carried out by Every Child Counts, during the summer term of 2008.

It was studied in ten schools in each of two LAs named 'A' and 'E'. The children were low attainers in Year 2, starting at an average National Curriculum level of just over level 1c. Results were assessed in terms of gains in National Curriculum points, with 0 representing 'Working toward level 1' and 7 representing level 3c; and in terms of standardised scores on the NFER standardised mathematics test. There were 41 children in LA A and 47 children in LA E. The children in LA A received intervention for longer than those in the other LAs in the Every Child Counts study, and received an average of 44 Mathematics Recovery lessons, whereas those in LA E received an average of 16 lessons.

In LA A, children progressed from an average score of 1.18 National Curriculum points at entry to 4.31 National Curriculum points at exit, giving a mean gain of 3.13 sublevels, and on average moving from level 1c to level 2c. In LA E, children progressed from an average score of 1.58 National Curriculum points at entry to 3.53 points at exit, showing a mean gain of 1.95 sublevels, and on average moving from level 1b to level 2c.

On the NFER standardised test, children in LA A progressed from a mean standard score of 71.92 to a mean standard score of 91.98 a mean gain of 19.26 points. Children in LA E progressed from a mean standard score of 75.00 to a mean standard score of 85.84: a mean gain of 10.84 points.

By the end of Year 2, 83% of the pupils in LA A had reached at least level 2c, and 41% had reached at least level 2b. 53% of the pupils in LA E (who had received less intervention) reached at least level 2c, and 32% reached at least level 2b.

4.2.2:2 Mathematics Recovery in Cumbria

Mathematics Recovery, as described above, was implemented in Cumbria in 2005 and is now used by at least 90 schools in the LA.

Children involved in the programme include children in Key Stage 1 who are working at level 1 or below; children at Key Stage 2 who are working at level 2 or below; and children at Key Stage 3 who are working at level 3 or below. The intervention is carried out by teachers and teaching assistants who have completed a CPD course provided by the LA and passed the LA accreditation criteria. Children receive three or four lessons per week, each lasting 20 or 30 minutes. The interventions are carried out one-to-one, with additional small group work in some cases.

Evaluation

During 2005/06 33 Year 1 pupils received funded Mathematics Recovery programmes. During 2006/07 28 Year 2 pupils received funded Mathematics Recovery programmes.

The children, who underwent the intervention in 2005-2007, were assessed before and after the intervention on the following components of Mathematics Recovery using the following tests.

Forward number word sequence

This assessment has six identified levels, from level 0 (unable to count in the range 0–10) to level 5 (able to count beyond 100 and identify 'one more' in this range). 81% of pupils made at least one level gain through the programme, and 4% gained three or four levels. 65% of pupils who made gains are now able to say number sequences between 30 and beyond 100 (level 2 National Curriculum).

Backwards number word sequence

This assessment has six identified levels, from level 0 (unable to count backwards in the range 0–10) to level 5 (able to count backwards from 100 and identify 'one less' in this range).

86% of the children made at least one level gain through the programme, and 21% gained three or four levels.

Numeral identification

This assessment has five identified levels, from level 0 (unable to identify any numerals) to level 4 (a child who can identify numerals to at least 1,000).

70% of children made at least one level gain, and 11% made at least three levels gain. The lower gains here than for the other components are due mainly to the children starting out at higher levels at the beginning.

Early arithmetical learning

This assessment has six identified levels of arithmetical development, from stage 0 (no addition or subtraction strategies, due to lack of one-to-one correspondence) to stage 5 (able to calculate using non-count by one strategies, such as rounding, doubling or using known facts).

91% of children made at least one level gain, and 29% made at least three levels gains.

Key Stage 1 Assessments

Of the 33 children who received funded intervention in Year 1, 70% reached at least level 2c, and 27% reached at least level 2b.

Of the 27 children who received funded intervention in Year 2, 50% reached at least level 2c, and 14% reached at least level 2b.

73% of the schools that took part in the study have shown improvement in their Key Stage 1 overall results.

The programme is also now being used in Key Stage 2, but results are not yet fully available.

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4.2.3 Individual Numeracy Support

This is a programme set up in Lancashire for young children (Year 1 and Year 2) who are failing to meet age-related expectations in mathematics. Children so far included in the programme have been those who were generally considered by schools as seriously behind in school work, and identified as having special educational needs (SEN).

Individual Numeracy Support is a one-to-one intervention for children who have significant gaps in their mathematical learning, or for whom group support has been attempted without success. It runs for 12 weeks including two weeks set aside for assessment, and involves three half-hour sessions per week. It is based on diagnostic assessments of children's individual needs and is carried out by staff trained in the programme: mostly teachers, but also including teaching assistants.

The scheme includes activities aimed at promoting visual and auditory memory, spatial awareness, and logical/sequential thinking as well as knowledge of number and the number system. The children undergo detailed assessment of numerous components of numeracy and of more domain-general cognitive processes, and they are given interventions targeted to their specific strengths and weaknesses. The numeracy-related components include reading, writing and ordering numbers; adding two numbers; recalling number bonds to ten; subtraction; and estimation. The other cognitive components include visual working memory; auditory working memory, spatial awareness and ability to visualise; logical/sequential thinking; perception; and processing information. Children are also given an attitude measure where they are asked to record on a scale how much they like mathematics and how good they are at the subject.

The Lancashire School Effectiveness Service run day training courses for SENCOs, teachers and teaching assistants involved in the programme.

Evaluation

So far, there has been a pilot study including ten children. Their average progress in National Curriculum points between the beginning and end of the intervention was 4.15 in Number; 3.47 in Shape, Space and Measures; and 2.99 in Using and Applying Mathematics.

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4.2.4 Mind the Gap, Devon (provisional title)

The target children are pupils in Year 6 who are performing somewhat but not severely below age expectations, and are at risk of not achieving level 4 in the Key Stage 2 National Curriculum tests.

The project is still in a relatively early stage and involves the use of diagnostic interviews and targeted interventions in the following areas:

1. Parts of the number system (positive whole numbers; integers including negative numbers; decimals);
2. Skills around these numbers (ordering and comparing; placing on a number line or scale; partitioning; equivalence; calculation);
3. Key skills, knowledge and understanding (understanding number – size of numbers, crossing boundaries, partitioning in different ways, numbers in different contexts; continuity of the number system – how numbers relate to one another, multiplying and dividing by ten, 100, etc., conversion in measures; mental calculation – using what you know to work out what you don't, choosing strategies).

Interventions are carried out with small groups, or sometimes individually. They are carried out in 'mirror rooms' (Trundle, 1998) where the sessions can be observed by other teachers and teaching assistants in an adjacent room through a one-way mirror. This can be used for training purposes.

The interventions involve weekly sessions of 20 to 30 minutes for ten weeks. The Devon team is writing materials which include linked sections of video, detailed session notes and resource ideas with clear guidance on key aspects of teaching and learning. These will also include diagnostic interview questionnaires.

Evaluation

The data is still in a raw state and have not been fully analysed. However, preliminary results suggest that 59% of the questions that children answered incorrectly in the first assessment were answered correctly in the final assessment. Teachers have reported that children are more willing to participate actively in maths lessons. More data will become available in the near future.

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4.2.5 Catch Up Numeracy

Catch Up Numeracy is a project set up in 2006-2007 by Catch Up, the independent trust that developed Catch Up Literacy. It is a development and expansion of Dr. Ann Dowker's Numeracy Recovery pilot project (Dowker, 2001, 2004) and is being funded by the Caxton Trust and the Esmee Fairbairn Charitable Trust.

It has been set up and tested in the following LAs: Brent, Hampshire, North Tyneside, Powys, Sandwell, the Vale of Glamorgan, Ealing, Norfolk, Stockton-on-Tees, Derby and Leeds. Typically, six schools from each LA were involved in the project; and during the research phase, each school identified two to four children to take part in the intervention, and two more children to act as controls.

Catch Up Numeracy was officially launched as a commercially available programme in July 2008. The Catch Up Numeracy intervention is currently used by approximately 160 schools and has been implemented in clusters of schools by approximately 27 LAs across England and Wales.

The target pupils for the intervention are pupils in Years 2 to 6 who have numeracy difficulties. As the programme is relatively non-intensive, it is expected to serve predominantly pupils with moderate mathematical weaknesses (e.g. those at level 1a or 2c on the Key Stage 1 National Curriculum tests) rather than the smaller number with extreme mathematical difficulties.

Catch Up Numeracy is based on ten components of numeracy and is delivered to the children by trained teachers or teaching assistants during two 15-minute sessions per week. The components addressed in the project are not regarded as an all-inclusive list of components of arithmetic, either from a mathematical or educational point of view. Rather, the components were selected because earlier research studies and discussions with teachers had indicated them to be important in early arithmetical development, and because research had shown them to vary considerably between individual children in the early school years (Dowker, 2004, 2005). The components that are the focus of the project include:

i. counting procedures;

ii. counting principles;

iii. ordinal numbers;

iv. use of written arithmetical symbolism;

v. use of place value in arithmetic;

vi. understanding and solution of word problems;

vii. translation between concrete and numerical formats;

viii. use of derived fact strategies for calculation;

ix. arithmetical estimation;

x. memory for number facts.

Each learner is assessed individually by a trained teacher/teaching assistant using 'Catch Up Numeracy formative assessments' which the member of staff then uses to complete the 'Catch Up Numeracy learner profile'. This personalised profile is used to determine the entry level for each of the ten Catch Up Numeracy components and the appropriate focus for numeracy teaching. Children are provided with mathematical games and activities targeted to their specific levels in specific activities. Where possible, these games and activities involve the use of materials that are commonly available in schools – including PNS Wave 3 materials, other published material and the standard apparatus of a primary mathematics classroom – rather than specially created apparatus.

Each 15-minute teaching session includes a review and introduction to remind the child of what was achieved in the previous session and to outline the focus of the current session: a numeracy activity, and a linked recording activity where the child records the results of the activity, in oral, written and/or concrete fashion, and where the child receives focused teaching related to their performance in the activity and any observed errors.

Children stay in the programme for one term, though teachers may adjust the length of intervention in the light of the pupils' progress.

Teachers and teaching assistants are given three half-day training sessions through a comprehensive Open College Network accredited training package for existing school-based teachers and teaching assistants; and are given a resource package including a book of documents and a DVD, including descriptions of the assessment techniques and of suggested activities.

Evaluation

154 children have so far been given the Basic Number Screening Test before and after entering the programme and their mathematics ages in months have been recorded. The gain in mathematics age was divided by the number of months elapsing between pre-test and post-test (between four and six months) to give a ratio gain. They have been compared with two control groups in the same schools: 42 children who received no intervention, and 50 who were given the same amount of time for individualised mathematics work, which usually involved reviewing work done in the school lessons and was not specifically targeted to assess strengths and weaknesses.

The 154 children who received Catch Up intervention showed a mean ratio gain of 2.2 (standard deviation 1.9).

The 50 children who received matched time intervention showed a mean ratio gain of 1.47 (standard deviation 1.78).

The 42 children who received no intervention showed a mean ratio gain of 1.25 (standard deviation 1.8).

The first two groups made a little more than the expected gain over time (suggesting some influence of the school being in the Catch Up Numeracy research and development project, since underachievers would be expected to make less than the expected gain over time). But those in the Catch Up Numeracy group made more than twice the expected gain. The results are highly significant. An analysis of variation, with Group as the factor and Ratio Gain as the dependent variable, showed a very significant effect of group ($F(2, 243) = 5.84$; $p < 0.01$) – so significant at the 0.01 level. The Catch Up intervention group made significantly higher ratio gains than either of the other groups, who did not differ significantly from one another.

Alan Evans of Cardiff University has acted as an independent evaluator and has reviewed the project at intervals (Evans 2007, 2008).

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4.3 Programmes involving the use of specific apparatus

Some schemes emphasise the use of particular forms of apparatus that are not routinely used in primary classrooms. The apparatus that is most commonly used is Numicon, sometimes in conjunction with other materials. Another type of apparatus is Addacus.

4.3.1 Numicon

Numicon was devised in 1996 as a form of apparatus that lends itself to multisensory teaching, integrating the auditory, visual, and kinaesthetic modalities. It was originally designed as a general teaching approach but was soon also applied to interventions with children with mathematical difficulties (Wing and Tacon, 1999; Ewan and Muir, 2002).

It was specifically designed to help children develop multisensory representations of number and includes Numicon shapes, cards which represent numbers with patterns of holes, which the child can fill with pegs. It also includes a 'feely bag', where the shapes can be placed for the child to identify by touch, and numeral cards, spinners, a display number line, and activity sheets for the teachers, indicating games and activities that can be used with the materials for counting, ordering, addition, subtraction and other mathematical activities. Numicon can be used for group activities for all children in a whole-class setting, or for individualised interventions with children who are experiencing difficulties.

Since, as in all teaching, success depends upon how materials are used, not upon materials themselves, misuse in poor teaching can lead to poor outcomes – see Hart (1989) for instance. Consequently there are a number of training and support options in place to facilitate effective use of Numicon, including a comprehensive Teaching Guide in each kit, a Training DVD and videos available on the website. In addition, there is a growing team of independent Numicon accredited trainers (currently 19 in number) and an increasing number of LA mathematics and/or Learning Support teams that have attended Numicon Training for Trainers (currently 26 LAs).

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4.3.1:1 Evaluation of Numicon within Every Child Counts Research Phase Study

Numicon was one of the interventions trialled for seven-year-olds with mathematical difficulties in the research carried out by Every Child Counts, during the summer term of 2008.

It was studied in ten schools in LA 'B'. 40 children were involved who were low attainers in Year 2, starting at an average National Curriculum level of 1b. They received an average of 27 intensive half-hour intervention sessions over one term. Results were assessed in terms of gains in National Curriculum points, with 0 representing 'Working toward level 1' and 7 representing level 3c; and in terms of standardised scores on the NFER standardised mathematics test.

The children progressed from an average score of 2.13 National Curriculum points at entry to 4.69 National Curriculum points at exit, giving a mean gain of 2.56 sublevels, and representing an average move from level 1b to level 2c. On the NFER test, they progressed from a mean standard score of 77.08 to a mean standard score of 94.89, a mean gain of 17.82 points. By the end of Year 2, 95% of the pupils had reached at least level 2c, and 63% had reached at least level 2b.

4.3.1:2 Visual models and images supported by signs and symbols in Brighton and Hove

This intervention is targeted at severely underachieving children in Year 2 to Year 4 who are performing at level 1 or below. It was observed that such children often have speech and language difficulties, and may be helped by extra use of visual images and kinaesthetic materials. The draft programme was launched in 2004, and involved the use of Numicon and Cuisenaire apparatus, together with use of sign and symbol.

The schools in the project were provided with Numicon and related materials, including Numicon plates, boards, number rods, number lines, spinners, numeral cards, feely bags and wall number line, activity cards and Makaton books.

The children in the programme are at School Action or School Action Plus on the SEN register; have a mathematics target on their individual education plans; and obtain low scores (less than 40 out of a possible 54) on a test based on the Early Maths Diagnostic Kit (by David and Margaret Lumb, available from NFER/Nelson cat. no. 4190 016).

Initially, it was piloted in five schools in 2004, and after children made good progress in the pilot study, was extended to other schools in the LA.

The programme involves the use of Numicon materials, supplemented by use of Makaton signs. It is carried out with small groups of no more than four children and involves three 20-minute sessions per week over ten weeks and is delivered by a teaching assistant trained in the programme. The teacher also uses the visual models and images and signs and symbols in whole-class teaching where appropriate.

Evaluation

In 2005, 60 teachers and teaching assistants from 25 Brighton and Hove schools attended training on the programme. Twelve of these schools have since sent in the data on their pre- and post-programme assessments. In these schools, all children made progress ranging from a gain of two to 38 marks (out of a total of 54). (The lower the pre-assessment score, the greater the gain that can be made.) The average gain was 14 marks.

Some schools achieved better results than others with similar starting scores.

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4.3.1:3 Visual models and images supported by signs and symbols in Bury

In 2007/08, Bury decided to adopt a programme similar to that of Brighton and Hove. The programme included 12 pupils from six schools who were all working below National Curriculum level 1 in numeracy at the end of Year 1. Pupils worked on the programme in the autumn term of Year 2.

At each school, a teacher and teaching assistant received a day's training from members of the Brighton and Hove team in use of the materials. The teaching assistant conducted the intervention daily over 12 weeks. The funding covered non-contact time within school for preparation and liaison.

Each school was provided with a full range of multisensory teaching equipment as in Brighton and Hove.

Evaluation

In Bury, the project is still in a pilot stage. However, the results so far appear positive. Pupils and teachers are very positive about the programme. The ten pupils who were part of the programme and who were given the same test as in Brighton and Hove improved from a mean score of 16.4 to a mean score of 27.1: a mean gain of 10.7 points. Two children were not given the test, but are known to have progressed from National Curriculum level W to levels 1c and 1b, respectively.

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4.3.1:4 Numicon in Leeds: Multisensory approach to the teaching and learning of mathematics

Leeds LA felt that some children who are underachieving in mathematics might benefit from the use of a multisensory approach, combining structured apparatus (Numicon) with number lines, visual memory games, whole-body gestures, etc.

Children in the programme are encouraged to handle the Numicon plates to help them to develop internal representations of numbers and their sequence. The distinctive ten-plate and 100-board apparatus are used to reinforce learning related to base ten. The teaching activities include games where children are instructed to visualise shapes, trace them, put them in order, move shapes about, etc., and explicitly require learners to visualise patterns or create a visual memory for a plate or pattern.

The programme also includes very clear, coloured number lines; themed room/wall displays relating to particular numbers and operations; and whole-body gestures for reinforcing operations (for example placing arms in + and = positions). For more advanced pupils, number tracks and rods are also used to facilitate learning arithmetic.

The original project included 29 pupils, aged eight to nine years, from nine schools. Training was provided for one teacher and one teaching assistant in each school.

Evaluation

Psychometric data were obtained for ten pupils before and after intervention, on British Abilities Scales subtests of specific cognitive skills (quantitative reasoning, visual and auditory memory and speed of processing) and on arithmetical performance (Basic Number Skills subtest of the British Abilities Scales and Wechsler Objective Numerical Dimensions).

Before intervention, the pupils performed poorly in visual memory (mean standard score 82) and extremely poorly in quantitative reasoning (mean standard score 50) but close to average in processing speed (mean standard score 90) and surprisingly well in auditory memory (mean standard score 140). Performance on both arithmetic tests was at floor level for most pupils at the beginning. After intervention, mean standard scores increased somewhat in both visual memory (from 82 to 89) and in quantitative reasoning (from 50 to 60). Raw scores increased significantly on both of the arithmetic tests, though floor effects made it impossible to study changes in standard scores.

Progress for all 23 pupils was monitored using PIVATS (an assessment that combines P levels with National Curriculum levels). PIVATS data were provided for July 2004 and December 2004, before the intervention, and for May 2005, five months after the intervention began. This showed little progress during the five months before the intervention (mean gains of 2.1 points for mathematics as a whole; 2.2 points for Number; and 1.6 points for Shape and Space) but considerable gains during the five months following the introduction of the materials (mean gains of 7.1 points for mathematics, 7.1 points for Number, and 7.4 points for Shape and Space). This corresponded on average to small gains within level 1a before the intervention, and a move from level 1a to level 2c after it. T-tests showed that the mean gains after the intervention were very significantly greater than those in a similar time period before the intervention ($p < 0.001$ in all cases).

Some pupils made much more significant progress than others following the multisensory mathematics programme. Interview and questionnaire data indicated that teachers and pupils liked the programme and the materials.

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4.3.2 Addacus Activity Packs, published by Addacus Ltd and used predominantly in Bradford

This is a multisensory numeracy intervention scheme, written by Celia Stone and Myra Nicholson, co-authors of the Beat Dyslexia Activity Packs. Addacus is designed to provide multisensory links (colour, touch, shape, talk skills, music and movement and poems) to enable early number concepts to be grasped and remembered. There are two books. Book 1, most relevant to interventions with primary pupils with difficulties, introduces numbers one to nine and zero; addition and subtraction; greater than and less than. It consists of 80 pages of activities and worksheets which involve the use of the plastic materials provided in the pack.

The pack contains the Addacus itself, an abacus designed to teach place value and the decimal number system. It features different-shaped posts which accommodate only nine cubes each, in order to teach place value with less potential for confusion about the difference between tens and units than the typical abacus. It also contains strings that vary in both length and colour, and cubes that can be strung on the strings to make string-alongs; number cards; number strips; plastic numbers; and a CD that provides number-related stories, songs, poems, and mental arithmetic activities.

The materials can be used with children of five-years-old and over who are experiencing difficulties in mathematics. They are designed to be used by teachers, teaching assistants or parent volunteers with a three-hour training session available. If necessary, they can be used without training as much of the instruction is explained on an audio CD and most of the instructions for tutors are on the page.

Intervention sessions last approximately 30 to 45 minutes per session and can be given daily, twice weekly, or as often as seems appropriate. It can be used with individuals or small groups.

Evaluation

There has been no formal evaluation yet, but comments by many users have been very positive, indicating that the children enjoyed them and made good progress with them.

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4.4 Schemes not fitting into the above categories

4.4.1 The Harrow Dyscalculia project

Although this project has some features of 3.2 (detailed individualised assessment) and 3.3 (use of multisensory materials), it stands out among most programmes in its explicit aim to identify and address dyscalculia specifically, as distinct from other causes of low achievement in mathematics.

The project involves pupils aged 8–14 in nine schools in Harrow. The pupils are given Butterworth's (2003) Dyscalculia Screener. The two tests within the screener used to identify dyscalculia are subitising (recognising, without counting, whether a number of randomly arranged dots matches a digit) and numerical Stroop (selecting which of two digits has the larger value, regardless of the physical size of the digits).

The pupils identified as dyscalculic are given individual support to help them improve their understanding of number. As they are assumed to lack the usual innately-programmed representations of numbers, a variety of concrete materials are used to help them build up such representations. These include dot patterns; bead strings; labelled and empty number lines; Cuisenaire rods and other base-10 apparatus; and nuggets (like flat-bottomed marbles). Teachers and teaching assistants are given training in helping children to use the materials and develop arithmetic strategies.

The project is ongoing, and a new associated project has been started by Brian Butterworth in association with the London Knowledge Lab to develop computerised versions of materials and activities that can be used online.

The outcomes have not yet been formally assessed as the study has not yet been completed; but teacher reactions have been positive.

For further information, see Messenger, Emerson and Bird (2007).

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4.4.2 The Jump Project in Lambeth

The JUMP programme (Junior Unrecognised Mathematics Prodigies) is a mathematics programme developed by John Mighton, a Canadian author and mathematician. This is a highly structured arithmetic programme, which includes a lot of direct instruction, repetition and reinforcement and starts with simplified models of problems, and then proceeds in small, incremental steps. The main materials are teaching manuals and pupil workbooks.

JUMP began as an individualised one-to-one programme for children having difficulties with mathematics, and has evolved into a comprehensive mathematics programme for children in Grades 3 to 7 in Canada.

In Lambeth, it is used for the original purpose of intervention for underachieving pupils. The target pupils are those in Years 5 and 6 who are expected not to reach level 4 in the Key Stage 2 National Curriculum tests, especially the most serious underachievers who are performing at least two years below age level. Interventions may be given either individually or in small groups. The programme involves a daily one-hour lesson for 30 days. Teachers and teaching assistants attend two days of training prior to the teaching programme.

During the 2006 summer term, 24 Lambeth primary schools participated in a pilot programme to evaluate the JUMP mathematics methods and materials. In the following year, the work continued and developed in 40 Lambeth primary schools. Schools that had participated in the pilot study were provided with a full set of all JUMP materials (Year 3 to Year 6) for teachers to adapt and use as they saw fit within the requirements of the National Schools Numeracy Strategy. Schools that had not participated in the pilot study were invited to re-run the pilot in January 2007.

In 2006/07, 30 schools were involved in the programme; 454 children (36% of the age cohort in the relevant schools) took part in the study; of these, 154 were two years or more below the expected level for their age.

Evaluation

In the pilot study in 2006, the children were given the 2003 version of the QCA optional test for Year 4 (maximum 55 marks), before and after the JUMP teaching programme: 90% improved their overall score; 58% improved by 6+ marks, 17% improved by 11+ marks and 6% by 16+ marks. Teachers also reported considerable improvements in pupils' attitudes and behaviour.

Of the 454 children participating in the main study of the JUMP programme in 2006/07, 33% achieved level 4 in the national tests at the end of Year 6. The performance of the JUMP children in the mental mathematics paper was consistently lower than their scores in the written papers might lead one to expect. Of the 154 children who were performing two years below age level (i.e. only at level 3c in Year 5), 69% were at or close to age-related expectations (level 4c or above) a year after starting the programme.

50% of the children made at least two National Curriculum sublevels of progress during the year, which is the typically expected level of progress for all pupils, and was a significant acceleration of these low-attaining pupils' progress in earlier years. Over a third (35%) of the children made three or more sublevels of progress, equivalent to one and a half school years or more in one year.

The children in the JUMP programme tended to show a discrepancy between written and mental mathematics papers, with better performance on the written paper. This may indicate that mental mathematics is intrinsically more difficult than written mathematics for low-attaining late primary school pupils, possibly because of working memory limitations. However, the LA has suggested that some schools and teachers may be over-relying on the workbooks, at the possible expense of developing speed and accuracy in mental mathematics, and has emphasised the need to use the mental mathematics 'warm up', with which most JUMP lessons are intended to begin.

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4.4.3 Talking Maths, Liverpool

This scheme was originally targeted at underperforming pupils in Years 1 to 3, though it has been extended to also include all of Key Stage 3. It involves work with small groups (typically three per group) who meet for three weekly sessions of approximately 30 minutes. The interventions are carried out by trained teaching assistants. Though the project was developed in Liverpool, it has also been used in Sheffield, Cheshire, Wigan, Lancashire, Islington and Sandwell.

This scheme focuses on teaching children to understand and use the language of mathematics.

Evaluation

So far, 32 pupils have been tracked to investigate changes in National Curriculum test performance between autumn and summer. Seven pupils made gains of three or more sublevels; 12 of two sublevels; 11 of one sublevel; and one made no progress.

One school sent a report describing the project very positively, and finding it particularly useful for children speaking English as an additional language.

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4.4.4 Year 1 Catch Up, Cumbria

(Note: This should not be confused with the Catch Up Numeracy scheme discussed above.)

This programme was devised by Nicky Cornfield, Viv Newby and Elaine Moran, Key Stage 1 staff at St Bridget's School, Egremont. It is provided by a teaching assistant to small groups of children in Year 1 who have been identified as being in need of some extra support to enable them to keep up with their peers.

The daily sessions each last for 20-25 minutes over a period of 12 weeks. There are four sessions written for each week. Lesson plans, vocabulary cards and resources have been prepared and are now kept in boxes so that they can be used every year.

Each session is broken into three parts: the first session is a mental maths 'starter', and the remaining two sessions are given questions as a title. The questions are displayed and read at the start of each part of the lesson, and the objective is explicitly discussed with the children: e.g. 'What are we going to learn?', 'To count up to ten things'.

The sessions involve training in counting, ordering, number recognition, addition to 20, subtraction from ten, and counting in sets larger than one: e.g. twos and fives. The activities become progressively more difficult, and include larger numbers, as the sessions progress. Apparatus used is that readily available in primary classrooms: various types of objects for counting; number lines and tracks and 100-squares.

Evaluation

No evaluation has yet been performed but the school found this programme sufficiently useful to wish to continue it in subsequent years.

Contact

http://numeracy.cumbriagridforlearning.org.uk

Appendix 1

Every Child Counts

Developing the programme

In its research and development phases, Every Child Counts is a partnership initiative between the Every Child a Chance charity (a coalition of business partners and charitable trusts), and government (DCSF and the National Strategies).

The aim is to enable the lowest-attaining child to make greater progress towards expected levels of attainment in mathematics, catching up with their peers and achieving level 2 or better and, wherever possible, level 2b or better by the end of Key Stage 1. The intention is to provide, from the academic year 2010/11, intensive support in mathematics to 30,000 Year 2 children annually. It is also expected that the highly trained teachers providing this support will have a wider impact on learning and teaching in their schools, and help to raise standards across the board.

Every Child Counts has three phases:

- research (academic year 2007/08);
- development (2008/09 and 2009/10);
- national roll-out (2010/11 onwards).

Research phase (2007/08)

The first phase of the initiative (2007/08) ran alongside, informed and was informed by the Williams review of the teaching of primary mathematics. Between September and December 2007 information was gathered through visits to different intervention programmes that are currently in place in LAs. This provided a baseline for further activity by summarising:

- the programmes/approaches in use;
- their training and support infrastructures;
- evidence of their impact on children's learning and progress.

Between January and July 2008, these existing models of intervention support were extended to new LA areas in order to identify the impact and draw out the essential features the national programme should incorporate to ensure success.

The five LAs involved were: Birmingham, Kent, Middlesbrough, Norfolk and Southwark. Ten schools were involved in each area. Additional small-scale work in Hackney tested out the impact of different periods of intensive support teaching. Information from the work in these authorities and schools informed the recommendations made in Chapter 4 of the Williams Report, *Independent Review of Mathematics Teaching in Early Years settings and primary schools.*
www.standards.dcsf.gov.uk/primary/mathematicsreview

Chapter 4 of this report, published in late June, addresses remit 3 from the Secretary of State to the review panel:

> *The review should specifically make recommendations to inform the development of an early intervention programme for children (age five to seven) who are failing to master the basics of numeracy – Every Child Counts – as recently announced by the Prime Minister.*

The content and recommendations of this section of the report reflect the findings of the ECC research phase as well as evidence given by providers and field visits by the panel to observe a wide range of intervention practice in schools. It is noted that some schemes have been developed by LAs and others by commercial organisations and that they differ in their reliance on a theoretical basis. It is emphasised that 'the recommendations of the report are to be seen in educational terms and do not constitute an endorsement of any specific products or services'. The panel did not consider that any single scheme exhibited all the features associated with a successful intervention and therefore, drawing on the review panel's observations and also evidence submitted to the review, asked that development of Every Child Counts (and particularly the intensive support element) should take account of the following essential features:

Recommendation 8: Intervention

The programme for intensive Wave 3 intervention in Every Child Counts should be based on the following characteristics:

1. *It should be led by a qualified teacher and should generally involve one child*
2. *However, the development phase of Every Child Counts should give adequate attention to assessing the benefits of small-group working, particularly in pairs*
3. *In assessing the child for intervention, the teacher with direct contact with the child must take the lead in shaping the decision to intervene; the use of video techniques in this and in training should be investigated further*
4. *Appropriate diagnostic tools should be developed to assist in assessment and in measuring progress on exit from intervention*
5. *Intervention in mathematics should be complete by the end of Key Stage 1; where a child needs intervention in both literacy and numeracy, both must be given equal priority over the course of Key Stage 1*
6. *A wide range of multisensory resources should be available to enable the child and the intervention specialist to select those appropriate to the specific circumstances*
7. *CPD programmes should be developed for both the intervention specialist and the LA intervention teacher leader*
8. *Consideration should be given to combining the roles of intervention specialist and Mathematics Specialist, depending on the size and circumstances of the school*
9. *Less intensive Wave 3 and Wave 2 interventions could be led by appropriately trained teaching assistants; consideration should be given to the training required and the use of interventions, with a robust evidence base of impact on learning and progress*
10. *A longitudinal study should be commissioned to assess the long-term benefits of intervention both at Key Stage 2 and, eventually, at GCSE level*

For further information about the outcomes of the Research Phase, see the reports posted on the LA section at www.everychildachancetrust.org

Development phase (2008/10)

Every Child Counts is based on the well-known 'three waves of intervention' model, which involves the development in schools of layers of support for children. The unique feature of Every Child Counts is the delivery of an intensive, teacher-led intervention for the lowest attaining child. This intervention approach is known as Numbers Count. In accordance with the steer from the Williams Report and the findings of the research phase, Edge Hill University, working in partnership with Lancashire LA, has taken the lead in developing Numbers Count.

The overall design of Numbers Count is for a 12-week programme, consisting of daily 30-minute sessions for the target children and delivered by the trained Numbers Count teachers. The core elements are a comprehensive diagnostic assessment of the child's strengths and weaknesses, core learning objectives for the lessons and guidance for teachers on lesson structure and key teaching approaches.

The areas of mathematics covered include:

- number and numeral recognition using the vocabulary of number – number words and connectives (before, more than, etc.);
- counting forward and back;
- counting in ones, twos and tens;
- mathematical vocabulary, use of language and pronunciation;
- understanding place value to partition and recombine numbers when calculating;
- ordering, sequencing and comparing by size;
- number bonds, addition and subtraction;
- arranging and rearranging to identify conservation of number and to begin to recognise patterns and relationships;
- practical examples of halving and doubling to identify inverse relationships;
- grouping and sharing to inform early understanding of multiplication and division;
- money including the use of real coins;
- problem solving in everyday contexts.

In the development phase the intensive Numbers Count intervention will be used in a growing number of clusters of LAs to prepare for nationwide roll-out. Over 2,500 children are expected to receive intensive teaching in the first year of the development phase and over 11,400 in 2009/10. It will be refined and further developed throughout the ECC development phase, 2008-10, by drawing on feedback from a wide range of participants and stakeholders as well as through the findings of an independent evaluation commissioned by the DCSF. Numbers Count teacher(s) and Teacher Leaders will be encouraged to contribute actively to the improvements so that Numbers Count can provide increasingly effective support for young children who have difficulties with mathematics.

Numbers Count aims to ensure the development of a numerate child through the work of a Numbers Count teacher teaching a Numbers Count lesson and supported by an Every Child Counts school. The intended outcome is to enable children who have the greatest difficulties with mathematics to make greater progress towards expected levels of attainment so that they will catch up with their peers and achieve level 2 or, where possible level 2b or better by the end of Key Stage 1.

The Every Child Counts programme aims to enable Numbers Count teachers to have a wider impact on learning and teaching in their schools and to help to raise standards across the board. The programme will expand beyond the core Numbers Count intervention to include less intensive interventions delivered by teaching assistants who receive ongoing support from the skilled Numbers Count teacher. The model used is one of 'layered interventions':

Wave 1	Quality First Teaching	All children
Wave 2	Small group additional intervention	Just below national expectations
Wave 3	Individual or very small group intervention with a trained and supported TA	Struggling
	Numbers Count Intervention on an individual and/or very small group basis with a trained specialist teacher	Lowest attaining

The purpose of the updated edition of 'What works for children with mathematical difficulties' is to inform guidance to schools on the effective interventions that can populate these 'layers'.

Roll-out (2010/11)

In 2010/11 the Every Child Counts programme will roll out nationally through the PNS so that 30,000 children will be reached in this first year of national implementation.

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