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Targeted interventions for children with arithmetical difficulties

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Background. Difficulty with arithmetic is a common problem. There is increasing evidence that arithmetical cognition is made up of multiple components, and that it is quite possible for children and adults to show strong discrepancies, in either direction, between the components. This suggested the desirability of developing interventions that assess and target children's weaknesses in specific components of numeracy.

Aims. This paper reports two related studies of targeted interventions for children with mathematical difficulties. The aim was to develop intervention techniques, based on assessing and targeting specific strengths and weaknesses, and to assess their effectiveness.

Sample. The first study, of the pilot Numeracy Recovery intervention programme, included 169 children aged 6 and 7. The second study, of the Catch Up Numeracy programme, included 246 children between the ages of 6 and 10, of whom 154 received the Catch Up intervention programme, 50 were given the same amount of time for non-targeted individualized mathematics work, and 42 children received no intervention, except for the usual school instruction. Both samples consisted of children, who had been identified by their teachers as having difficulties with arithmetic.

Method. The first study took place in Oxford, and children were assessed on nine components of early numeracy. The project has since been adapted, under the name of Catch Up Numeracy, for wider use, and tested in 11 local authorities in the UK. The second study, which included 10 components of numeracy, involved teachers and teaching assistants receiving formal training from the Catch Up organization in delivering the programme. In both studies, following assessment, the children received weekly intervention (half an hour a week for approximately 30 weeks) in the components with which they had difficulty.

The children were given standardized tests at the beginning and end of intervention. In the first study, the tests used were the Basic Number Skills subtest of the *British Abilities Scales*; the Numerical Operations subtest of the *Wechsler Objective Numerical Dimensions*; and the Arithmetic subtest of the *Wechsler Intelligence Scale for Children*. In the second study, the test used was the *Basic Number Screening Test*.

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Results. The children in the first study showed significant improvement, relative to their age group, on the three standardized tests. In the second study, the children in the Catch Up intervention programme made more than twice as much progress in 'mathematics age' as would have been expected by the passage of time. Their progress was significantly greater than that of either of the control groups.

Conclusions. The findings support the view that individualized interventions in arithmetic, especially those that focus on the particular components with which an individual child has difficulty, can be highly effective. Moreover, the amount of time given to such individualized work does not, in many cases, need to be very large to be effective. Thus, children's arithmetical difficulties are highly susceptible to intervention.

Up till now, there has been much less research on mathematical development and difficulties than on some other areas of development, such as language and literacy. However, there has recently been an increased emphasis on mathematics in cognitive developmental research (e.g. Baroody & Dowker, 2003; Campbell, 2005; Royer, 2003); in neuroscience (Ansari, Garcia, Lucas, Hamon, & Dhilil, 2005; Butterworth, 1999; Dehaene, 1997); and in educational policy and practice in the UK and abroad (Gross, 2007; Kilpatrick, Swafford, & Fundell, 2001; Williams, 2008).

Difficulty with arithmetic is a common problem. There is a significant overlap between difficulties in reading and mathematics. The overlap between diagnoses of dyslexia and dyscalculia has ranged from 20 to 60% in different studies (Butterworth & Yeo, 2004). However, there is definite evidence that mathematical difficulties are not simply a result of dyslexia, other language difficulties, or generally low IQ, as mathematics and reading difficulties can sometimes dissociate. For example, Gross (2007) points out that in the 2005 Key Stage 2 SATS, 5.9% of Year 6 pupils were achieving below level 3 in mathematics, 6.3% in English, and only 3.9% in both. This finding indicates that it is quite possible for a child to have severe problems in either mathematics or English without the other.

Gross' study supports earlier findings from several countries about specific mathematical difficulties, e.g. Lewis, Hitch, and Walker (1994) in England; Gross-Tsur, Manor, and Shalev (1996) in Israel; and Bzufka, Hein, and Neumarker (2000) in Germany. These studies suggest that about 6% of children have severe specific difficulties with arithmetic. A far higher number of children have difficulties that are less severe or less specific, but which still cause significant practical, educational, and later employment difficulties. This figure depends in part on the demands for numeracy in a particular society at a given time but is likely to be at least 15–20% of the population (Bynner & Parsons, 1997).

There is increasing evidence that arithmetical cognition is made up of multiple components, and that it is quite possible for children and adults to show strong discrepancies, in either direction, between the components. This has been found for typically developing children (Dowker, 1998, 2005); children with arithmetical difficulties (Butterworth, 2005; Denvir & Brown, 1986; Geary & Hoard, 2005; Russell & Ginsburg, 1984; Temple, 1994); typical adults (Geary & Widaman, 1992; LeFevre & Kulak, 1994); and patients with dyscalculia resulting from brain damage (Butterworth, 1999; Dehaene, Piazza, Pinel, & Cohen, 2003; Delazer, 2003; Warrington, 1982; Zamarian, Lopez-Rolon, & Delazer, 2007). Most recently, functional brain imaging studies have supported the view that different aspects of mathematical processing can involve different areas and circuits in the brain (Castelli, Glaser, & Butterworth, 2006; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Gruber, Indefrey, Steinmetz, & Kleinschmidt, 2001; Kaufmann, 2008; Rickard *et al.*, 2000).

There are different views as to which components, and what levels of distinction between components, are most important in the study of mathematical development. Some researchers make the broad distinction between factual, conceptual, and procedural knowledge (e.g. Delazer, 2003; Greeno, Riley, & Gelman, 1984). Even within these types of knowledge there can be more specific dissociations: e.g. between procedures for different arithmetical operations; or between use of concepts in derived fact strategies and in estimation (Dowker, 2005, 2007; Jordan, Mulhern, & Wylie, 2009).

Though different components of arithmetic often correlate with one another, weaknesses in any one of them can occur relatively independently of weaknesses in the others. Weakness in even one component can ultimately take its toll on performance in other components, partly because difficulty with one component may increase the risk of the child relying exclusively on another component, and failing to perceive and use relationships between different arithmetical processes and problems; and partly because when children fail at certain tasks, they may come to perceive themselves as 'no good at maths' and develop a negative attitude to the subject. However, *components of arithmetic do not form a strict hierarchy*. A child may perform well at an apparently difficult task (e.g. word problem solving) while performing poorly at an apparently easier component (e.g. remembering the counting word sequence). Several studies (e.g. Denvir & Brown, 1986) have suggested that it is not possible to establish a strict hierarchy whereby any one component invariably precedes any other.

The present paper is divided into two parts. The first part describes a pilot intervention programme, which is based on the findings that suggest that arithmetic is made up of numerous components, and involves assessing and targeting individual children's specific weaknesses. The second part discusses the development and modification of the project for much wider use.

The interventions are based on an analysis of the particular subskills which children bring to arithmetical tasks, with remediation of the specific areas where children show problems. The components addressed here are not to be regarded as an all-inclusive list of components of arithmetic, either from a mathematical or educational point of view. Rather, the components were selected because earlier research studies (e.g. Dowker, 1998) and discussions with teachers had indicated them to be important in early arithmetical development, and because research has shown them to vary considerably between individual children in the early school years (Denvir & Brown, 1986; Dowker, 1998, 2005).

Part 1: Numeracy Recovery scheme

The Numeracy Recovery scheme has been discussed earlier (e.g. Dowker, 2001, 2005, 2007). It was piloted with 6- and 7-year-olds (mostly Year 2) in six first schools in Oxford, and was funded by the Esmée Fairbairn Charitable Trust. The scheme involved working with children who have been identified by their teachers as having problems with arithmetic.

Participants

A total of 168 children were included in the study. They included 77 boys and 91 girls. Their mean age was 80.7 months ($SD = 6.1$).

Procedure

These children were assessed on nine components of early numeracy, which are summarized and described below. The children then received weekly individual intervention for half an hour in the particular components with which they had been found to have difficulty. The interventions were mostly carried out by the classroom teachers, though in some schools teaching assistants were also involved.

The teachers were released (each teacher for half a day weekly) for the intervention, by the employment of supply teachers for classroom teaching. Each child remained in the programme for 30 weeks, or until their teachers felt that they no longer needed intervention; whichever was shorter.

Components that formed the focus of the project

(1) Counting procedures

Arguably the most basic component of arithmetic is the ability to make appropriate use of counting. While most 6-year-olds have achieved relatively effortless counting, a significant number have not (Griffin, Case, & Siegler, 1994; Yeo, 2003). This may seriously impede their development of arithmetic, both because of the intrinsic logical relationships between counting and arithmetic, and because the effort of counting may distract attention from other aspects of arithmetic (Gray & Tall, 1994; Yeo, 2003).

In the pre-test, children were tested for:

(a) Accuracy of counting sets of 5, 8, 10, 12, and 21 objects;
(b) Rote verbal counting to 10 and 20

Children were assessed as needing intervention if they made errors in counting 12 objects, or in counting verbally to 20.

Intervention. Children practised counting sets of objects, ranging in number from 5 to 25. Counting procedures were a problem only for a small but important minority of the children. About 5% received intervention in this component.

(2) Counting-related principles and their application

Gelman and Gallistel (1978) proposed five main principles that guide counting. There is much debate as to whether these principles precede, follow, or develop in tandem with counting procedures; but it is generally agreed that most children have acquired all the counting principles before the age of 5 or 6. However, the order irrelevance principle (that counting the same set of items in different orders will result in the same number) is usually the latest of the main counting principles to be acquired (Cowan, Dowker, Christakis, & Bailey, 1996); and is sometimes weak even in 6-year-olds. Understanding the order irrelevance principle appears to be related to the ability to predict the result of adding or subtracting an item from a set (Cowan *et al.*, 1996; Dowker, 2005).

The only one of Gelman and Gallistel's counting principles to be directly addressed was the order irrelevance principle; this is the only principle likely to be still absent in children over six in a mainstream school, even with mathematical difficulties. The ability to predict the results of repeated additions and subtractions by one was also assessed. This is viewed here as an integral part of a full understanding of the counting principles, as (a) a genuine understanding that the order of counting is irrelevant to number is only possible in the context of an awareness that some counting-related changes *are* relevant to number; and (b) it represents the first understanding of the relation of counting to arithmetic.

In the pre-test for the 'counting principles' component, children were assessed on:

(a) The order irrelevance principle. The children watched an adult count a set of objects, and are then asked to predict the result of further counts:
  - in the reverse order;
  - after the addition of an object; and
  - after the subtraction of an object.

(b) Repeated addition by 1. Children saw a set of five items, and were then shown one more item being added, and asked to say, without counting, how many there are now. This was repeated up to 15.

(c) Repeated subtraction by 1. Children saw a set of 10 items, and were then shown one item being subtracted, and asked to say, without counting, how many there are now. This was repeated down to zero.

Children were assessed as needing intervention if they made any errors in any of these tasks.

Intervention. For the order irrelevance principle, children practised counting and answering order-irrelevance questions about very small numbers of counters (up to four), where the numerosity of the set is likely to be obvious to the child. During this practice, the adult made statements such as, 'It's four this way, and four that way – it's four whichever way you count it!'. The child was then given further practice with increasingly large sets.

For repeated addition by 1 and repeated subtraction by 1, children were given practice in observing and predicting the results of such repeated additions and subtractions with counters (up to 20). They were then given verbal 'number after' and 'number before' problems: 'What is the number before 8?', 'What is the number after 14?', etc. They were given some worksheets devised for the project, including repeated addition and subtraction by one from a set of circles.

About 16% of the children received intervention in this component.

(3) Written symbolism for numbers

Children often experience difficulties with representing quantities as numerals (Fuson, 1992; Ginsburg, 1977). With regard to this component, children were asked to read aloud a set of 18 single-digit and two-digit numbers. They also wrote a similar set of numbers to dictation.

If children made more than two errors in either the reading or writing task, they were assessed as needing intervention.

Intervention. Children practised reading and writing numbers. Children with difficulties in reading or writing two-digit numbers (tens and units) are given practice in sorting objects into groups of 10, and recording them as '20', '30', etc. They were then given such sorting and recording tasks where there were extra units as well as the groups of 10.

About 34% of the children received intervention in this component.

(4) Understanding the role of place value and using tens and units in number operations and arithmetic

The understanding of place value and associated ability to carry out multidigit arithmetic are known to be a source of difficulty for many children (e.g. Fuson & Burghardt, 2003;

Hiebert & Wearne, 1994; Thompson, 2003). In this study, children were assessed on eight sums that involved adding tens to units (e.g. $20 + 3 = 23$); eight sums that involved adding tens to tens (e.g. $20 + 30 = 50$); and eight sums that involved combining the two into one operation ($20 + 33 = 53$). A related test involved pointing to the larger number in 20 pairs of two-digit numbers, that varied either just with regard to the units (e.g. 23 vs. 26); just with regard to the tens (e.g. 41 vs. 51); or where both tens and units varied in conflicting directions (e.g. 27 vs. 31; 52 vs. 48).

Children were assessed as needing intervention if they made more than two errors in any of the addition tasks, or more than four errors in the number comparison task.

Intervention. Children were shown the addition of tens to units and the addition of tens to tens in several different forms: (a) written numerals; (b) number line or number block; (c) hands and fingers in pictures; (d) 10-pence pieces and pennies; and (e) any apparatus (e.g. multilink or unifix) with which the child is familiar. The adult emphasized the fact that these give the same answers.

Children, whose difficulties were more specific to the use of place value in arithmetic, were given practice with arithmetical patterns such as: '$20 + 10$; $20 + 11$; $20 + 12$', etc.; being encouraged to use apparatus when necessary.

About 77% of the children received intervention in this component.

(5) Word problem solving

There is a considerable body of evidence (Hughes, 1986; Mayer, 2003; Riley, Greeno, & Heller, 1983) that young children often experience difficulty with word problems in arithmetic, even when they are capable of performing the necessary calculations. The semantic characteristics of the problems have a strong influence on how easily they are solved. For example, children tend to find problems involving changes in quantity ('change' problems; e.g. 'Joe had seven sweets and he ate four; how many did he have left?') easier than those involving comparisons between quantities ('compare' problems; e.g. 'Joe has seven sweets and Tom has four; how many more sweets does Joe have than Tom?'; De Corte & Verschaffel, 1987; Riley *et al.*, 1983).

As a pre-test, children were given the Word Problems Test devised by Griffin *et al.* (1994) for addition and subtraction. This consists of five sums, which include the presentation of 'change' and 'compare' problems for both addition and subtraction and of 'combine' problems (combining two quantities) for addition. These are usually regarded as the most common and important types of word problems (e.g. De Corte & Verschaffel, 1987). Children were assessed as needing intervention if they made more than two errors.

Intervention. Children who had difficulties in understanding word problems were presented with short addition and subtraction word problems of 'change', 'compare', and 'combine' types (similar but not identical to those used in the assessments). The adult discussed the problems with them using probes such as 'What are the numbers that we have to work with?', 'What do we have to do with the numbers?', 'Do you think that we have to do an adding sum or a taking-away sum?', and 'Do you think that John has more sweets or fewer sweets than he used to have?'. They were encouraged to use counters to represent the operations in the word problems, as well as writing the sums numerically.

About 58% of the children received intervention in this component.

(6) Translation between arithmetical problems presented in concrete, verbal, and numerical formats

Hughes (1986) reported that many primary schoolchildren demonstrate difficulty in translating between concrete and numerical formats (in either direction), even when they are reasonably proficient at doing sums in either one of these formats and has suggested that this difficulty in translation may be an important hindrance to children's understanding of arithmetic.

The translation pre-test involves six types of task, for both addition and subtraction. The tasks involved:

(a) Translation from numerical to concrete, where children were presented with two sums and were invited to 'show how to do this sum with the counters'.
(b) Translation from concrete to numerical, where they watched the researcher perform two arithmetical operations with counters and were asked to write down the sum that the researcher did.
(c) Translation from verbal to concrete, where children were presented with six word problems, and invited to 'show me this story with the counters'.
(d) Translation from verbal to numerical, where they were presented with six word problems, and asked to 'write down the sum that goes with the story'.
(e) Translation from numerical to verbal, in which they saw two written sums, and were asked to 'tell me a story that can go with this sum'.
(f) Translation from concrete to verbal, where they watched the researcher perform two arithmetical operations with counters, and were asked to 'tell me a story to go with what I'm doing here'.

A scoring system was devised (Dowker, 2005, 2007), based on whether for each item children gave a complete response (representing the numbers and the operation; score 3); a near-complete response (representing the numbers and the operation, but omitting an aspect such as the equals sign, or converting a subtraction problem into its inverted addition form; score 2); an incomplete response (representing just one or more of the numbers; score 1) or an incorrect response (producing unrelated numbers, a sum involving the wrong operation, or not responding at all). As there were 20 problems in all, the maximum possible score was 60. Children were considered to need intervention if they scored 24 or less.

Intervention. The techniques involved in intervention for this component focused specifically on showing that the same arithmetical problem can be represented in different ways. The children were shown the same problems in different forms (problems similar but not identical to those used in the pre-test); and were shown that they gave the same results. They were also encouraged to represent word problems and concrete problems by numerical sums, and to represent numerical problems and word problems by concrete objects.

They also played 'Same or Different' games, where the experimenter presented a problem in one form (e.g. '$6 + 2$') and then demonstrated its correct representation (e.g. two counters being added to six counters) or incorrect representation (e.g. two counters being taken away from six counters). They were asked to say whether the second problem was the same or different from the first problem.

About 62% of the children received intervention with this component.

(7) Derived fact strategies in addition

One crucial aspect of arithmetical reasoning is the ability to derive and predict unknown arithmetical facts from known facts, for example, by using arithmetical principles such as commutativity, associativity, and the addition/subtraction inverse principle (Baroody, Ginsburg, & Waxman, 1983; Canobi, Reeve, & Pattison, 1998, 2003; Dowker, 1998, 2005). For example, if we know that $9 + 3 = 12$, we can use the commutativity principle to derive the fact that $3 + 9$ is also 12.

In the *pre-test* to assess this component, children were given the Addition and Subtraction Principles Test developed by Dowker (1998). In this test, they were given the answer to a problem and then asked to solve another problem that could be solved quickly by the appropriate use of an arithmetical principle (e.g. they may be shown the sum '$23 + 44 = 67$' and then asked to do the sum $23 + 45$, or $44 + 23$). Problems preceded by answers to numerically unrelated problems were given as controls. The children were asked whether 'the top sum' helps them to do 'the bottom sum', and why. The actual addition and subtraction problems varied in difficulty, ranging from those which the child could readily calculate mentally, through those just beyond the child's calculation capacity, to those very much too difficult for the child to solve without using the related problem. There were five principles tested: identity (e.g. if $8 + 6 = 14$, $8 + 6$ must be 14); commutativity; adding 1 to an addend results in adding 1 to the sum; subtracting 1 from an addend results in subtracting 1 from the sum; and the addition-inverse principle. Children who did not use at least two of the principles were considered to need intervention.

Intervention. Intervention techniques for this component involved training in the use and application of derived fact strategies. The children were presented with pairs of arithmetic problems similar to those used in the pre-test. The 'derived fact strategy' techniques were pointed out and explained to them; and the children were invited to solve similar problems. If they failed to do so, the strategies were demonstrated to them for single-digit addition and subtraction problems, with the help of concrete objects, and of a number line; and they were again invited to carry out other derived fact strategy problems.

About 71% of the children received intervention in this component.

(8) Arithmetical estimation

The ability to estimate an approximate answer to an arithmetic problem, and to evaluate the reasonableness of an arithmetical estimate, are important aspects of arithmetical reasoning (LeFevre, Greenham, & Waheed, 1993; Siegler & Booth, 2005; Sowder & Wheeler, 1989).

Pre-test. This involves the estimation task previously used by Dowker (1997, 2005). Children were presented with a series of addition problems, and with estimates made for these problems by imaginary characters (Tom and Mary). The children were asked (a) to evaluate Tom and Mary's estimates on a five-point 'smiley faces' scale from 'very good' to 'very silly'; and (b) to suggest 'good guesses' for these problems themselves. Children were given an initial mental calculation test, and then given estimation problems within the range just too difficult for them to solve by mental calculation. Each child was given nine estimation problems. Reasonable estimates were defined as estimates that were within 30% of the correct answer and were also higher than the larger addend. Children who gave four or fewer reasonable estimates were considered to need intervention.

Intervention. Children were given additional 'Tom and Mary' evaluation tasks, and were asked to give reasons for their answers; and further practice in producing their own estimates. This was done under several conditions:

(a) arithmetical problems similar to those used in the pre-test;
(b) problems with small numbers using concrete objects; and
(c) word problems that provided a realistic practical context for estimation.

About 56% of the children received intervention in this component.

(9) Number fact retrieval

Knowledge of number facts contributes to efficiency in calculation (Tronsky & Royer, 2003), and is a significant factor in distinguishing between mathematically normal and mathematically 'disabled' children (Geary & Hoard, 2005; Jordan & Hanich, 2000; Ostad, 1998; Russell & Ginsburg, 1984).

Pretest. This was based on Russell and Ginsburg's (1984) Number Facts Test, which assessed retrieval of 10 basic addition facts, ranging in difficulty from $2 + 5$ to $9 + 8$. Children who made more than five errors were considered to need intervention.

Intervention. Children were presented with some basic addition and subtraction facts (e.g. $3 + 3 = 6$; $6 + 6 = 12$). As suggested, for example, by Ginsburg (1977), the main technique was to ask the child to do the same sums repeatedly (during the same session, and in successive sessions), in the hope that the repetition would lead to retention of the facts involved. If the child continued to carry out the same problem over and over again as though it were a new problem, the child was explicitly asked, 'Have we done this sum before?', 'What did we get?', 'Do you think you can tell me what the answer will be, before you work it out.'

They also played 'number games' (e.g. some from Straker, 1996) that reinforce number fact knowledge.

About 67% of the children received intervention in this component.

Evaluation of effectiveness

The children in the project were given three standardized arithmetic tests: the Basic Number Skills subset of the *British Ability Scales*, 2nd edition (BAS; Elliott, Smith, & McCulloch, 1996); the Numerical Operations subtest of the *Wechsler Objective Numerical Dimensions* (WOND; Wechsler, 1996); and the Arithmetic subtest of the *Wechsler Intelligence Scale for Children*, 3rd edition (WISC-III; Wechsler, 1991). The first two placed greatest emphasis on computation abilities and the latter on arithmetical reasoning. The children were retested 6 months after beginning intervention.

Results

The children's initial scores on the standardized tests, and their retest scores after 6 months, were analysed. The children in the intervention group made significant improvements on all the standardized tests according to matched-pairs *t* tests. The mean standard scores on the BAS Basic Number Skills subtest were 95.53 ($SD = 11.66$) initially and 100.25 ($SD = 12.46$) after approximately 6 months ($t = -5.38$, $df = 167$, $p < .01$). The mean standard scores on the WOND Numerical Operations test were 90.45 ($SD = 10.95$) initially and 92.67 ($SD = 12.42$) after 6 months ($t = -2.75$, $df = 167$, $p < .01$). The mean standard scores on the WISC Arithmetic subtest were 6.9 ($SD = 2.91$) initially and 8.37 ($SD = 2.5$) after 6 months ($t = -6.24$, $df = 167$, $p < .01$).

Part 2: Further development as Catch Up Numeracy

This project has been adapted (under the title Catch Up Numeracy) for wider use in collaboration with Julie Lawes and Wayne Holmes of Catch Up (Caxton Trust). The key aim in this further development was to make it possible for the programme and the assessment materials to be constructed and presented in a form that could be used readily by teachers and teaching assistants, after training of a level that was affordable in terms of both time and money. In addition, some of the components of the programme were adapted, so as to be applicable across a wider range of children. For example, while relatively few children in mainstream schools have difficulty with basic counting procedures (5% in the pilot study needed intervention with this component), more have difficulty with more difficult counting tasks, such as backward counting and counting by 2s, and such tasks were incorporated into this stage of the project.

Training programmes and materials have been developed for use by teachers and teaching assistants. The target pupils for the intervention are pupils in Years 2–6 who have numeracy difficulties. Schools have selected whichever children they feel were likely to benefit; but the main target group are children with moderate mathematical weaknesses (e.g. those at level 1a or 2c on the Key Stage 1 SATs) rather than the smaller number with extreme mathematical difficulties.

Children in the project receive interventions from trained teachers or teaching assistants during two 15 min sessions per week. The components addressed in the project are not regarded as an all-inclusive list of components of arithmetic, either from a mathematical or educational point of view.

Each child is assessed individually by a trained teacher/teaching assistant using 'Catch Up Numeracy formative assessments' which the member of staff then uses to complete the 'Catch Up Numeracy learner profile'. This personalized profile is used to determine the entry level for each of the 10 Catch Up Numeracy components and the appropriate focus for numeracy teaching. Children are provided with mathematical games and activities targeted to their specific levels in specific activities. Where possible, these games and activities involve the use of materials that are commonly available in schools, rather than specially created apparatus. These include the standard apparatus of a primary mathematics classroom, readily available published materials, and the Wave 3 materials that are currently provided to schools by the British government for use by children with difficulties in mathematics. The latter include a variety of objects, cards, games, and activities provided to schools. For example, they include cards showing numerals in units, tens, hundreds, and thousands; fraction symbols; symbols for operations; vertical and horizontal number lines; number tracks; sets of pictures to match with numerical symbols; tape measures; beads and strings; cubes; spinners; dice and dominoes.

Each 15-min teaching session includes a review and introduction to remind the child of what was achieved in the previous session and to outline the focus of the current session; a numeracy activity; and a linked recording activity where the child records the results of the activity, in oral, written, and/or concrete fashion, and where the child receives focused teaching related to their performance in the activity and any observed errors.

Children stay in the programme for one term; though teachers may adjust the length of intervention in the light of the pupils' progress.

Teachers and teaching assistants are given three half-days' training through a comprehensive Open College Network CN accredited training package for existing school-based teachers and teaching assistants; and are given a resource package including a book of documents and a DVD, including descriptions of the assessment techniques and of suggested activities.

The programme includes 10 components of numeracy, 9 of which are slightly adapted versions of the 9 components described in Part 1 of the paper. The tenth is the understanding of ordinal number.

The components and, where relevant, their subcomponents are as follows:

(1) counting verbally (subcomponents: counting verbally from 0 or 1; counting on from a given number; counting back from a given number);
(2) counting objects (subcomponents: counting objects; order irrelevance; repeated addition of objects; repeated subtraction of objects);
(3) reading and writing numbers (subcomponents: reading numerals; reading number words; writing numerals);
(4) hundreds, tens, and units (subcomponents: number comparison; adding tens and units; subtracting tens and units);
(5) ordinal numbers: the ordinal task involves showing children pictures of bead strings and telling them that the bead with the dot in the middle is the first (or in some pictures tenth) bead. One bead in each picture is shaded, and the children are asked to say which bead it is (e.g. second, fourth, ninth, etc.);
(6) word problems;
(7) translation (subcomponents: translating from objects to numerals; translating from numerals to objects; translating from number words to numerals; translating from number words to objects);
(8) derived fact strategies;
(9) estimation; and
(10) remembered number facts.

The programme typically involves two 15-min teaching sessions, rather than a single half-hour teaching session as in the project described in Part 1 of the paper. Each session involves focused teaching on one particular component.

Table 1. Catch Up Numeracy levels, national curriculum levels, maths ages, and number ranges

Catch Up Numeracy level	National curriculum level	Approximate maths age	Catch Up Numeracy number range
1	P7 (working towards P8)	$<$ 5; 6	1–5
2	P8	$<$ 5; 6	0–8
3	P8 (working towards level 1)	$<$ 5; 6	0–10
4	Level 1c	5; 6	0–15
5	Level 1b	6; 0	0–18
6	Level 1a	6; 6	0–20
7	Level 2c	7; 0	0–50
8	Level 2b	7; 6	0–80
9	Level 2a	8; 0	0–100
10	Level 3c	8; 6	0–500
11	Level 3b	9; 0	0–800
12	Level 3a	9; 6	0–1,000

Formative assessments are carried out before starting the individual teaching sessions. The formative assessments take 2–3 h in total, but are administered in small blocks of time. A learner profile is constructed, showing the child's level of performance in each of the components. The levels of performance within each component are assigned mainly on the basis of the range of numbers with which the children can deal. Table 1 shows the Catch Up Numeracy levels, applicable to each component. For each Catch Up Numeracy level, there is a number range, which is based on the expectations of the Primary Numeracy Strategy. The number range enables the teaching assistant or teacher to control the level of difficulty being introduced by the size of the numbers used. This enables the children to tackle new concepts within a number range, with which they are comfortable. As the children gain confidence and competence with the concept, the number range is extended.

Initially, the focus of the teaching sessions is on components where the child falls below Catch Up Numeracy level 3. Once they have achieved this level, the focus is on getting the components up to Catch Up Numeracy level 6; then level 9; and finally level 12.

A variety of activities are provided, many similar to those in the original Numeracy Recovery project. In addition links are made to relevant Primary Numeracy Strategy and Wave 3 materials.

The project was set up and tested in some schools (2–6 per authority) in each of 11 local authorities in England and Wales. Each school identified 2–4 children to take part in the intervention, and two more children to act as controls.

Participants

The participants were 246 primary schoolchildren. One hundred and fifty-four of these received the Catch Up intervention programme. Fifty were given the same amount of time for individualized mathematics work, which usually involved reviewing work done in the school lessons and was not specifically targeted to assessed individual strengths and weaknesses. Forty-two children received no intervention, except for the usual school instruction.

They were all given Gillham and Hesse's (2008) Number Screening Test before and after the intervention. This test was chosen because it can be administered by school staff, unlike the tests used in the pilot study, which can only be used by, or under the direction of, trained psychologists. Also, it assesses skills considered important in the current British mathematics curriculum. Most of the assessments were carried out by school staff, with some being carried out by research assistants Peter Morris and Chongying Wang.

The Catch Up intervention group had a mean age of 104.43 months ($SD = 14.95$). Their mean British Abilities Basic Number Skills standard score was 88.61 ($SD = 13.31$). Their mean score on the Number Screening Test was 12.08 ($SD = 16.15$) and their mean percentile was 86.2 ($SD = 10.23$). Their mean mathematics age was 89.92 months ($SD = 13.07$).

The matched time group had a mean age of 110.11 months ($SD = 13.49$). Their mean score on the Number Screening Test was 12.79 ($SD = 15.37$) and their mean percentile was 87.15 ($SD = 11.33$). Their mean mathematics age was 91.9 months ($SD = 10.81$).

The no-intervention group had a mean age of 105.78 months ($SD = 12.79$). Their mean score on the Number Screening Test was 14.47 ($SD = 16.15$) and their mean percentile was 88.61 ($SD = 13.31$). Their mean mathematics age was 92.88 months ($SD = 12.48$).

ANOVAs with group (Catch Up intervention vs. matched time controls vs. no-intervention controls) as the grouping factor revealed no significant group differences in age, number screening score, percentile, or mathematics age.

Results

The ratio gain (months gained in mathematics age on Gillham and Hesse's Number Screening Test, divided by the number of months between initial and final testing) was investigated. The 154 children who received Catch Up intervention showed a mean ratio gain of 2.2 ($SD = 1.9$); i.e. they made more than twice as much progress as would be expected from passage of time alone. The 50 children who received matched time intervention showed a mean ratio gain of 1.47 ($SD = 1.78$). The 42 children who received no intervention showed a mean ratio gain of 1.25 ($SD = 1.8$).

An ANOVA, with group as the factor and ratio gain as the dependent variable, showed a very significant effect of group, $F(2, 243) = 5.84, p < .01$). Tamhane's T2 *post hoc* tests showed that the Catch Up intervention group made significantly higher ratio gains than either of the other groups, which did not differ significantly from one another.

Discussion

The findings suggest that individually targeted interventions are effective for children with mathematical difficulties, and may work better than similar amounts of individual attention in mathematics that are not targeted to a child's specific strengths and weakness. In the Catch Up study, both control groups made somewhat more than the expected gain over time (suggesting *some* influence of the school being in the Catch Up Numeracy research and development project, since underachievers would be expected to make less than the expected gain over time). But those in the Catch Up Numeracy group made more than twice the expected gain. The results are very highly significant.

Thus, the findings so far appear to support the view that any extra help in arithmetic is likely to give some benefit; but that interventions that focus on the particular components with which an individual child has difficulty are likely to be more effective than those which assume that all children's arithmetical difficulties are similar. Moreover, the amount of time given to such individualized work does not, in many cases, need to be very large to be effective: the children in both the Numeracy Recovery pilot study and the Catch Up study received approximately half an hour a week, and showed considerable benefits.

The studies also indicated that it was possible to adapt the project for use in a wide variety of regions of the UK, and with a wider age range than had been initially used. It also showed that it was relatively easy to train teachers and teaching assistants to use the intervention successfully.

More generally, the research strongly supports the view that children's arithmetical difficulties are highly susceptible to intervention. It is not the case that a large number of children are simply 'bad at maths', and that this cannot be ameliorated; though further research is still needed to establish the degree to which such improvements are maintained. It is particularly notable that in the pilot study described in Part 1, some of the greatest improvement occurred in the WISC Arithmetic subtest: a test sometimes regarded as a measure of predominantly 'innate' intelligence.

In the last few years, there has been increased interest in the UK in individually targeted interventions (Dowker, 2009); and several intensive programmes have been developed, e.g. the Hackney Numeracy Recovery project; the Mathematics Recovery programme originally devised in Australia (Wright, Martland, & Stafford, 2005); and some programmes using the Numicon apparatus (e.g. Devon Primary Maths Team, 2006). In 2008, the Every Child Counts programme was set up with the aim of developing and providing individualized interventions for all children with mathematical difficulties in Year 2.

It is important to carry out further research comparing the effectiveness of different interventions, and comparing all of them with similar amounts of individual attention. In this context, it is important to investigate the *relative* effectiveness of interventions for children with different characteristics: e.g. age on entering programmes; level and type of arithmetical difficulties; and performance on other cognitive tests such as those measuring verbal and spatial ability.

Although the focus in this paper is on intervention as such and its effects, there are many possibilities for bidirectional relationships between research and intervention. Research, and the resulting conclusions about arithmetical development and its components, inspired the intervention project. Similarly, studying the characteristics of children selected for intervention, and the effects of the intervention, can also serves to test the theories about arithmetical development and its components (Dowker, 2007).

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